Drude Theory of Metals

1897 - electron discovered by J. J. Thomson
→ 1900 - Drude theory

Electrons considered as a gas inside the metal

Valence electrons → conduction electrons
of atom
of metal

Total charge \(-eZ\)

Electrons move almost freely against background of
heavy, immobile charged ions

electron density \(n = \frac{N}{V}\)

\(N_A\) = Avogadro's number \(= 6.022 \times 10^{23}\) (particles/mole)

\(\rho_m = \text{mass density} \ (g/cm^3) = M\)
\(A = \text{atomic mass} \ (\text{grams/mole})\)

\(n = \frac{\text{grams}}{\text{cm}^3} \times \frac{\text{moles}}{\text{gram}} \times \frac{\text{atoms}}{\text{mole}} \times \frac{\text{# conduction electrons}}{\text{atom}}\)

\(= \rho_m \times \frac{1}{A} \times N_A \times Z\)

\(= 6.022 \times 10^{23} \frac{Z \rho_m}{A}\)
\[ N = \frac{N}{V} \text{ typically } 10^2 - 10^3 \text{ e/cm}^3 \]

⇒ Volume of a conduction electron

\[ V_e = \frac{V}{N} = \frac{1}{N} = \frac{4}{3} \pi \Gamma_s^3 \]

⇒ \[ \Gamma_s = \left( \frac{3}{4 \pi N} \right)^{\frac{1}{3}} \]

\[ \Gamma_s \text{ values typically } 1 - 3 \, \text{Å} \quad (\text{Å} = 10^{-10} \text{ m}) \]

**Basic Assumptions**

1. Collisions neglected
   - Independent electron
   - Free electron
   - approximations

2. Collisions instantaneous
   - ? ? do the electrons bump off the ion core ?

3. Collision time \( \tau \), relaxation time, mean free time
   - probability per unit time \( \frac{1}{\tau} \)

[drunk men analogy: bumps into something]

\[ \text{every 10 minutes} \]

& in 1 hour, Prob # of bumps = 60 mm x \( \frac{1}{10 \text{ min}} \)

= 6
Basic Assumptions Cont.

4. th. equl. achieved via collisions

After collision, electron's velocity is \( \vec{V}_0 \)
randomized with respect to velocity before

\[
\vec{V}_0 = \vec{V}_{\text{after}} = \mathcal{F}(\vec{V}_{\text{before}})
\]

\[\text{Drude} \rightarrow \text{Ohm's Law } V = IR\]

\[
\vec{E} = \rho \frac{\vec{J}}{I}
\]

\( \rho \) = resistivity

\[
\vec{J} = -ne \vec{V}
\]

\( \vec{V} \) = average electron velocity

\( \text{no field } \Rightarrow \vec{V} = 0\)

\[
\vec{V} = \mathcal{F}(\vec{V}_0)
\]

\[
\vec{V} = \mathcal{F}(\vec{E})
\]

\[
\vec{V} = \text{average} \left( -\frac{e\vec{E}t}{m} \right)
\]

\[
\vec{V} = -\frac{e\vec{E}t}{m} \Rightarrow \vec{J} = \frac{ne^2}{m} \vec{E}
\]

\[
= \sigma \vec{E}
\]
\[ \tau = \frac{n e^2 \tau}{m} \]

\[ j = \sigma E \]

obtain relaxation time by measuring the resistence of a metal

\[ \tau = \frac{m \sigma}{n e^2} = \frac{m}{\rho n e^2} \]

\[ \tau = \frac{4 \pi r_0^3}{3} \frac{1}{\rho} \frac{m}{e^2} \quad \text{with} \quad \alpha_e = \frac{e^2}{m e^2} \]

\[ \Rightarrow \tau = \frac{0.27}{\rho_n} \frac{r_0^3}{\alpha_e^3} \times 10^{14} \text{ sec} \]

\[ l = v_o \tau \]

? can we use \[ \frac{1}{2} m v_e^2 = \frac{3}{2} k_b T \] Classical Equi

Energy

\[ \Rightarrow v_e \approx 10 \text{ cm/s at 300K} \]

\[ \Rightarrow l = 1-10 \text{ Å} \]

But this \( v_e \) is about 10x too small

And at low Ts, \( T \) is about 10x larger than at 300K
$\varnothing$ $\rho$ can be very large!

up to $1$ cm in special samples
at low $T$'s

$\Rightarrow$ do electrons "bump" all of ions recently?

so how do we predict $\varnothing$?

$\Rightarrow$ $\varnothing$ - independent quantities "highly valued"