Last time, we discussed how that for a metal-vacuum-metal tunnel junction, the tunnel current will be a function of both the applied tunnel voltage $V$ and the localized density of states of the sample at the Fermi level, $\rho_s(S, E_f)$.

$$I \propto V \rho_s(S, E_f)$$

$\rho_s(S, E_f)$ means it's the LDOS at $E_f$ and at the position of the tip.

![Diagram](image)

Again, the LDOS of the sample is defined by:

$$\rho_s(Z, E) = \frac{1}{\varepsilon} \sum_{E_n = E - \varepsilon}^{E_f} |\psi_n(Z)|^2$$

Chen, Intro. to STM, p.6.

which is related to the square of the wavefunction summed over the states within the energy window $E-\varepsilon \rightarrow E$. 
In the following, we continue to refer to the book "Introduction to Scanning Tunneling Microscopy" by C. Julian Chen, Oxford.

Note in the above expression, that we have not talked about any "scanning." This is only an expression for two electrodes separated by a distance $s$.

When we "scan" the tip over the surface, it is usually done in "constant current" mode.

Let's imagine a real surface now, in which the LDOS varies with position at the "surface." Actually, how do you define where the "surface" is?

\[ \Delta z = \text{contour of constant electron probability density} \]

If $I \propto V_p(s, \xi_p)$, the the tip needs to follow this contour of constant electron probability density in order to maintain the constant current. Then the height "corrugations" are approximately proportional to the electron probability density, i.e. $p_{\xi}$. 
This is because the probability density falls off
\[ \rho_s \sim \psi(z) \sim e^{-2kz} \]
approximately linear over small range

Thus in this range, we have:
\[ \rho_s \sim \Delta z \]

The tip height corrugations relate to the local density of states of the sample surface.

This description is, however, not quite the whole story (by any means). We have not taken into account the tip's density of states. Although if the \( \rho_t \) is constant, we find it will be just a constant factor in the equation.
According to Bardeen's theory of tunneling between 2 metal electrodes, the tunneling matrix element is given by a surface integral over a separation surface between the two electrodes.

\[ M = \frac{i}{2\hbar} \int_{z=z_0} (\psi^* \frac{\partial \psi}{\partial z} - \psi \frac{\partial \psi^*}{\partial z}) \, ds \]

- Chen, p. 22

\( \psi \) and \( \psi^* \) are wavefunctions of the two electrodes.

Fermi's Golden Rule gives the probability \( W \) of an electron in state \( \psi \) at energy \( E_\psi \) to tunnel to a state \( \chi \) at energy \( E_\chi \):

\[ W = \frac{2\pi}{\hbar} |M|^2 \delta(E_\psi - E_\chi) \]

- Chen, p. 22

only states at same energy can tunnel into each other
The current can be written as:

\[ I = \frac{4\pi e}{h} \int_{-\infty}^{\infty} \left[ f(E_F - eV + \varepsilon) - f(E_F + \varepsilon) \right] \rho_s(E_F - eV + \varepsilon) \rho_T(E_F + \varepsilon) d\varepsilon \]

Fig. 1.20. The Bardeen approach to tunneling theory. Instead of solving the Schrödinger equation for the coupled system, a, Bardeen (1960) makes clever use of perturbation theory. Starting with two free subsystems, b and c, the tunneling current is calculated through the overlap of the wavefunctions of free systems using the Fermi golden rule.

\[ f = \frac{1}{1 + e^{(E - E_F)/kT}} \]
If the energy window (or resolution) is larger than $hT$, then the Fermi functions can be approximated by step functions:

\[ f(E) \]

\[ f_s(E) \]

\[ f^T(E) \]

Then the integral reduces to:

\[ I = \frac{4 \pi e}{h} \int_0^{eV} \rho_s(E_F - eV + \epsilon) \rho_F(E_F + \epsilon) d\epsilon \]

\[ I \approx \frac{C \pi e}{h} \int_0^{eV} \rho_s(E_F - eV + \epsilon) \rho_F(E_F + \epsilon) d\epsilon \]

Chen, p. 23