Drude Theory of Metals

1897 - electron discovered by J. J. Thompson

→ 1900 - Drude theory

electrons considered as a gas inside the metal

valence electrons of atom → conduction electrons of metal

total charge $-eZ$

electrons move almost freely against background of heavy immobile charged ions

electron density $n = \dfrac{N}{V}$

N_A = Avogadro's # = 6.022×10^{23} (particles / mole)

ρ_m = mass density (g/cm³) = M

A = atomic mass = grams/mole

$$n = \frac{g}{cm^3} \times \frac{moles}{gram} \times \frac{atoms}{mole} \times \frac{\#\text{ conduction electrons}}{atom}$$

$$= \rho_m \times \frac{1}{A} \times N_A \times Z$$

$$= 6.022 \times 10^{23} \frac{Z \rho_m}{A}$$

$$n = \frac{N}{V} \quad \text{typically } 10^{22} - 10^{23} \ e/cm^3$$

$\Rightarrow$ volume of a conduction electron

$$v_e = \frac{V}{N} = \frac{1}{n} = \frac{4}{3}\pi r_s^3$$

$$\Rightarrow \quad r_s = \left(\frac{3}{4\pi n}\right)^{1/3}$$

r_s values typically 1 - 3 Å (Å = 10^{-10} m)

Basic Assumptions

1. collisions neglected

   independent electron } approximations
   free electron }

2. collisions instantaneous

   ?? do the electrons bump off the ion cores?

3. collision time τ, relaxation time, mean free time

   probability per unit time $1/\tau$

[drunken man analogy: bumps into something every 10 minutes]

in 1 hour, Prob # of bumps = $60 \text{ min} \times \left(\frac{1}{10 \text{ min}}\right)$

$= \underline{6}$

Basic Assumptions Cont.

4. th. equil. achieved via collisions

after collision, electron's velocity is $\vec{v}_0$
randomized with respect to velocity before

$$\vec{v}_0 = \vec{v}_{after} \neq f(\vec{v}_{before})$$

Drude $\rightarrow$ Ohm's Law $V = IR$

$$\vec{E} = \rho \vec{j}$$

ρ = resistivity

$$\vec{j} = -ne\vec{v}$$

$\vec{v}$ = average electron velocity

no field $\Rightarrow$ $\vec{v} = 0$

$$\vec{v} \neq f(\vec{v}_0)$$

$$\vec{v} = f(\vec{E})$$

$$\vec{v} = \text{average}\left(-\frac{e\vec{E}t}{m}\right)$$

$$\vec{v} = -\frac{e\vec{E}\tau}{m} \Rightarrow \vec{j} = \frac{ne^2\tau}{m}\vec{E}$$

$$= \sigma \vec{E}$$

$$\sigma = \frac{n e^2 \tau}{m}$$

$$\vec{j} = \sigma \vec{E}$$

obtain relaxation time by measuring the resistance of a metal

$$\rightarrow \quad \tau = \frac{m\sigma}{n e^2} = \frac{m}{\rho n e^2}$$

$$\tau = \frac{4\pi r_s^3}{3} \frac{1}{\rho} \frac{m}{e^2} \quad \text{with} \quad \boxed{a_0 = \frac{\hbar^2}{m e^2}}$$

$$\Rightarrow \boxed{\tau = \frac{0.22}{\rho_\mu} \frac{r_s^3}{a_0^3} \times 10^{-14} \text{ sec}}$$

HW #1 Problem 3: Show

$$l = v_0 \tau$$

? can we use

$$\frac{1}{2} m v_0^2 = \frac{3}{2} k_B T$$

Classical Equipartition of Energy

$$\Rightarrow v_0 \simeq 10^7 \text{ cm/s at } 300\text{K}$$

$$\Rightarrow l = 1 - 10 \text{ Å}$$

But this v_0 is about 10x too small

And at low T's, τ is about 10x larger than at 300K

$\Rightarrow$ λ can be very large!

up to ~1 cm in special samples
at low T's

$\Rightarrow$ do electrons "bump" off of ions really?

so how do we <u>predict</u> τ ?

$\Rightarrow$ τ-independent quantities "highly valued"