Drude Theory (cont.)

Since $\vec{v}(t) = \dfrac{\vec{p}(t)}{m}$

$$\vec{j} = -ne\frac{\vec{p}(t)}{m}$$

We therefore need an equation for time evolution of $\vec{p}(t)$

$\vec{p}(t)$ is the momentum per electron

In the absence of collisions, we would have:

$$\vec{p}(t+dt) = \vec{p}(t) + \vec{f}(t)dt \implies \frac{d\vec{p}(t)}{dt} = \vec{f}(t)$$

But since there is a probability of collisions $\frac{dt}{\tau}$

The probability of no collision is $1 - \frac{dt}{\tau}$

This reduces the momentum at $t + dt$

$$\implies \vec{p}(t+dt) = \left(1 - \frac{dt}{\tau}\right)\left[\vec{p}(t) + \vec{f}(t)dt\right]$$

$$\implies \boxed{\frac{d\vec{p}(t)}{dt} = -\frac{\vec{p}(t)}{\tau} + \vec{f}(t)}$$

Application to case of electric + magnetic fields

$$\rightarrow \quad \frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{p}}{mc} \times \vec{H}\right) - \frac{\vec{p}}{\tau}$$

For steady state, this gives a system of equations which satisfy

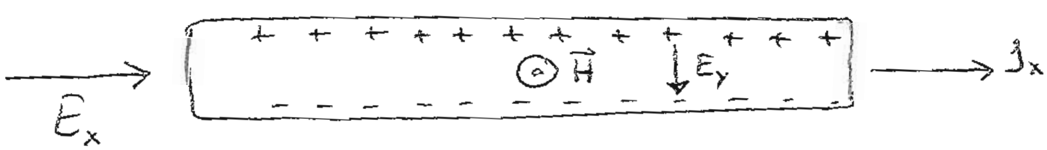
$$\frac{d\vec{p}}{dt} = 0$$

Hall Effect

An example of above is the case of an applied electric field in x-direction + an applied magnetic field in z-direction.

Lorentz force

$$-\frac{e}{c}\,\vec{v} \times \vec{H}$$



results in an E_y

magnetoresistance

$$\rho(H) = \frac{E_x}{j_x}$$

Hall coefficient

$$R_H = \frac{E_y}{j_x H}$$

$\hat{x} \times \hat{y} = \hat{z}$

$\hat{y} \times \hat{z} = \hat{x}$

$\hat{z} \times \hat{x} = \hat{y}$

↳ $\hat{x} \times \hat{z} = -\hat{y}$

$$0 = -eE_x - \overbrace{\left(\frac{eH}{mc}\right)}^{\omega_c} p_y - \frac{p_x}{\tau}$$

$$0 = -eE_y + \overbrace{\left(\frac{eH}{mc}\right)}^{\omega_c} p_x - \frac{p_y}{\tau}$$

$$\omega_c = \frac{eH}{mc}$$

$$0 = \frac{ne^2\tau}{m} E_x + \frac{ne\omega_c\tau}{m} p_y + \frac{ne}{m} p_x$$

$$0 = \sigma_0 E_x - \omega_c \tau j_y - j_x$$

$$\boxed{\sigma_0 E_x = \omega_c \tau j_y + j_x}$$

$$0 = \frac{ne^2\tau}{m} E_y - \frac{ne\omega_c\tau}{m} p_x + \frac{ne}{m} p_y$$

$$0 = \sigma_0 E_y + \omega_c \tau j_x - j_y$$

$$\boxed{\sigma_0 E_y = -\omega_c \tau j_x + j_y}$$

$\frac{d\vec{p}}{dt}$ equation leads to:

$$\sigma_0 E_x = \omega_c \tau j_y + j_x$$

$$\sigma_0 E_y = -\omega_c \tau j_x + j_y$$

For $j_y = 0$,

$$\sigma_0 E_x = j_x \Rightarrow \rho(H) = \frac{1}{\sigma_0}$$

and $\sigma_0 E_y = -\omega_c \tau j_x =$

$$\sigma_0 = \frac{ne^2\tau}{m} \; ; \; \omega_c = \frac{eH}{mc}$$

$$\Rightarrow \quad E_y = -\frac{\omega_c \tau}{\sigma_0} j_x = +\left(\frac{-1}{nec}\right) H j_x$$

$$\Rightarrow \boxed{R_H = -\frac{1}{nec}}$$

sign (+ or −) of R_H then depends on

sign (+ or −) of <u>e</u>

we said by definition e is a positive number

then for electrons as charge carriers, R_H is <u>neg.</u>

but what if R_H is measured to be <u>+ (pos.) ?</u>

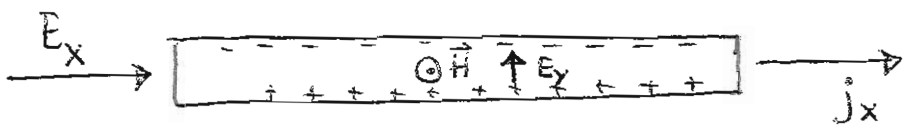
How do you measure R_H ?

$$E_y = R_H \, H \, j_x$$

$$\therefore \quad R_H = \frac{E_y}{H \, j_x}$$

sign of R_H depends on sign of E_y

If one has positive charge carriers,



$q \, \vec{v} \times \vec{B}$ or $+ \frac{e}{c} \vec{v} \times \vec{H}$

is "down" in this picture

"pos charge" accumulates on the bottom side

$\Rightarrow$ E_y points "up"

For $j_y \neq 0$ but then $E_y = 0$

$$\Rightarrow \qquad \tan\phi = \frac{j_y}{j_x}$$

ϕ = Hall angle

$$0 = -\omega_c \tau \, j_x + j_y$$

$$\frac{j_y}{j_x} = \omega_c \tau = \text{Hall angle}$$

<u>if</u> $\omega_c \tau$ is small, ϕ is small

$\Rightarrow$ electron can only complete a small part of a revolution between collisions

<u>If</u> $\omega_c \tau$ is large, ϕ is large

$\Rightarrow$ electrons complete larger fraction of the cyclotron orbit before collision