AC Electrical Conductivity of a Metal

Can a metal be transparent?

$\Rightarrow$ very small, or no, absorption
very high transmission

again, start with Drude result

$$\left.\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} + \vec{f}\ \right\} \qquad \text{Eq. [redacted]}$$

this time we have an oscillatory electric field

$$\vec{E}(t) = \mathrm{Re}\left(\vec{E}(\omega)\, e^{-i\omega t}\right)$$

and $\quad f = -e\vec{E}$

with $\quad \vec{p} = \mathrm{Re}\left(\vec{p}(\omega)\, e^{-i\omega t}\right)$

also $\quad \vec{j}(t) = \mathrm{Re}\left(\vec{j}(\omega)\, e^{-i\omega t}\right) = -ne\dfrac{\vec{p}}{m}$

So $\quad -i\omega\,\vec{p}(\omega) = -\dfrac{\vec{p}(\omega)}{\tau} - e\vec{E}(\omega)$

$$\left(-i\omega + \frac{1}{\tau}\right)\vec{p} = -e\vec{E}$$

$$\Rightarrow \vec{j} = -\frac{ne}{m}\left(\frac{-e}{-i\omega + 1/\tau}\right)\vec{E}$$

$$\vec{j} = \frac{ne^2}{m}\left(\frac{1}{1/\tau - i\omega}\right)\vec{E}$$

$$\vec{j}(\omega) = \sigma(\omega)\,\vec{E}(\omega)$$

$$\sigma(\omega) = \left(\frac{\tau}{\tau}\right)\frac{\frac{ne^2}{m}}{1/\tau - i\omega} = \frac{\sigma_0}{1 - i\omega\tau}$$

$$\sigma_0 = \frac{ne^2\tau}{m}$$

Maxwell's Equations

- use them to get the appropriate wave equation

start with

$$\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t}$$

$$\rightarrow \quad \nabla \times (\nabla \times E) = -\frac{1}{c}\,\nabla \times \frac{\partial H}{\partial t}$$

$$-\nabla^2 E = \frac{i\omega}{c}\,\nabla \times H$$

$$= \frac{i\omega}{c}\left[\frac{4\pi}{c}\,j + \frac{1}{c}\frac{\partial E}{\partial t}\right]$$

$$= \frac{i\omega}{c}\left[\frac{4\pi\sigma}{c}\,E - \frac{i\omega}{c}\,E\right]$$

$$-\nabla^2 E = \frac{\omega^2}{c^2}\left[1 + \frac{4\pi i\sigma}{\omega}\right]E$$

$$-\nabla^2 E = \frac{\omega^2}{c^2}\left[\varepsilon(\omega)\right] E$$

$$\varepsilon(\omega) = 1 + \frac{4\pi i}{\omega}\left[\frac{\sigma_0}{1 - i\omega\tau}\right]$$

$$= 1 + \frac{4\pi i}{\omega}\left[\frac{\sigma_0/\omega\tau}{1/\omega\tau - i}\right]$$

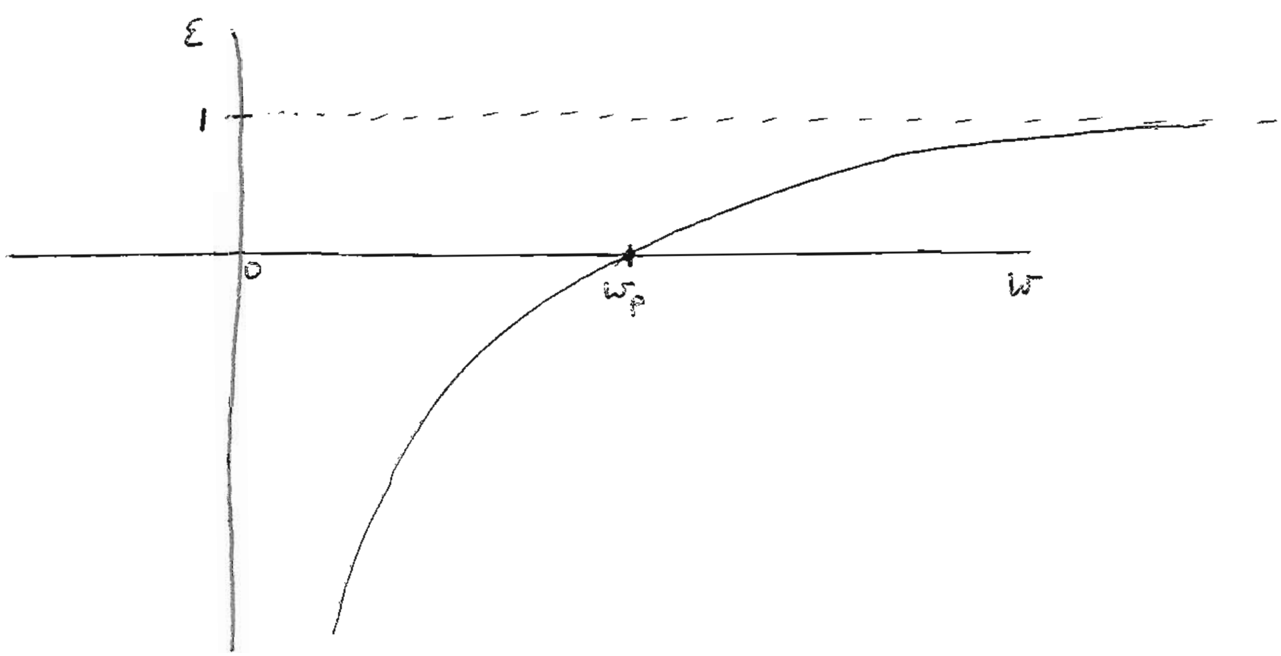
let $\omega\tau \gg 1$

$$\Rightarrow$$

$$\varepsilon(\omega) \simeq 1 - \frac{4\pi}{\omega^2}\frac{\sigma_0}{\tau}$$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\omega_p^2 = \frac{4\pi\sigma_0}{\tau} = \frac{4\pi}{\cancel{\tau}}\frac{ne^2\cancel{\tau}}{m} = \frac{4\pi ne^2}{m}$$



For $\omega < \omega_p$, $\varepsilon < 0$ (negative)

($\text{let } \varepsilon = -1$ for example)

$$\Rightarrow \quad -\nabla^2 E = -\frac{\omega^2}{c^2} E$$

$$\Rightarrow \quad \nabla^2 E = \frac{\omega^2}{c^2} E$$

$$\text{let } E = E_0 e^{-ax}$$

$$\Rightarrow \quad a^2 E_0 e^{-ax} = \frac{\omega^2}{c^2} E_0 e^{-ax}$$

$$\Rightarrow \quad a^2 = \frac{\omega^2}{c^2}$$

$\Rightarrow$ solution is

$$\left. E(x) = E_0 e^{-\frac{\omega}{c} x} \right\} \text{decaying exponential}$$

For $\omega > \omega_p$, $\varepsilon > 0$ (positive)

(let $\varepsilon = \varepsilon_p$ where ε_p is positive)

$$\Rightarrow \quad -\nabla^2 E = \frac{\omega^2}{c^2} \varepsilon_p E$$

Solution:

$$E = E_0 e^{i(kx-\omega t)}$$

$$\Rightarrow \quad +k^2 \cancel{E} = \frac{\omega^2}{c^2} \varepsilon_p \cancel{E}$$

$$\Rightarrow \quad +k^2 = \frac{\omega^2}{c^2} \varepsilon_p$$

$$k = \frac{\omega}{c} \sqrt{\varepsilon_p}$$

Oscillatory Solution

For waves of frequency $\omega > \omega_p$, they can propogate.

$\Rightarrow$ transparent metal

Charge Density Oscillations

Continuity Equation

$$\nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$$

$$\nabla \cdot j(\omega) = i\omega \rho(\omega)$$

$$= \nabla \cdot \sigma E(\omega) = \sigma \, \nabla \cdot E = \sigma (4\pi \rho)$$

$$\Rightarrow \quad i\omega \rho(\omega) = 4\pi \, \sigma(\omega) \rho(\omega)$$

$$\Rightarrow \quad i\omega = 4\pi\sigma(\omega)$$

$$\Rightarrow \quad i\omega - 4\pi\sigma(\omega) = 0$$

$$\Rightarrow \quad 1 + \frac{4\pi i \sigma(\omega)}{\omega} = 0$$

same condition as 1.35 but set $= 0$

can have a CDW if

$$\boxed{M\ddot{d} = -Fd}$$

F = Force

M = Total Mass

$M = Nm$

$F = NeE = Ne\,4\pi\sigma$

σ = surface charge density = nde

nde = electron density × displacement × electron charge

$$= \frac{e^-}{cm^3} \times cm \times \frac{\text{charge}}{e^-}$$

$$= \frac{\text{charge}}{cm^2}$$