Thermal Conductivity : Drude model

Drude model's most impressive success (at the time) was the explanation of the Wiedemann-Franz law

Wiedemann-Franz law :

$$\frac{\text{thermal conduct.}}{\text{electrical conduct.}} = \frac{K}{\sigma} \propto T \quad \Big\} \text{ for many metals}$$

$$\frac{K}{\sigma T} = \text{proportionality constant}$$

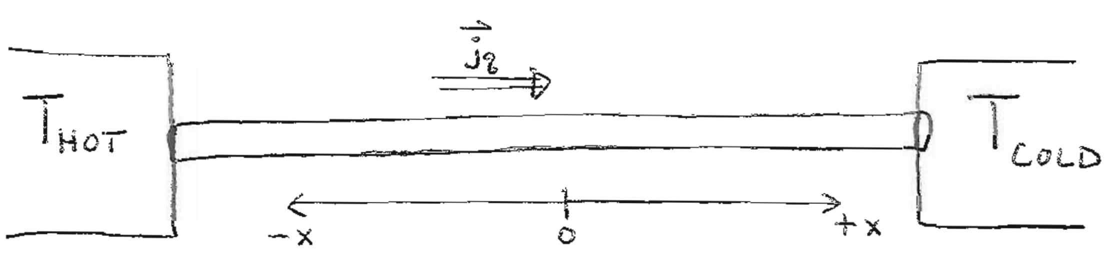
$$\frac{K}{\sigma T} = \text{Lorenz number} = L$$

experimentally,

$$L \simeq 2.2 \times 10^{-8} \text{ watt-ohm}/K^2$$

Drude explanation

assumption : bulk of $\overbrace{\text{thermal current}}^{\vec{j}_q}$ in a metal carried by the conduction electrons



$$\frac{dT}{dx} = - \text{ (negative)}$$

$$j_q = + \qquad\qquad \vec{j}_q = -K\,\nabla T$$

Goal: is to use Drude model to calculate the Lorenz number

through some basic concepts, one can get:

1-D $$j_q = \frac{1}{2} n v \left[\mathcal{E}(T[x - v\tau]) - \mathcal{E}(T[x + v\tau]) \right]$$

$$= \frac{\text{elec.}}{\text{cm}^3} \times \frac{\text{cm}}{\text{s}} \times \text{energy}/\text{electron} \quad \Big\} \text{ dimensional analysis}$$

$$j^q = \frac{\text{energy}}{\text{cm}^2 \cdot \text{s}} = \text{energy flux}$$

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j^q can be written as:

$$j^q = -\frac{1}{2} n v \,(2v\tau)\, \frac{\left[\mathcal{E}(T[x + v\tau]) - \mathcal{E}(T(x - v\tau)) \right]}{(2v\tau)}$$

$$= n v^2 \tau \, \frac{d\mathcal{E}}{dx} \quad \text{where } 2v\tau \Rightarrow dx$$

$$j^q = n v_x^2 \tau \, \frac{d\mathcal{E}}{dT} \left(-\frac{dT}{dx} \right)$$

$$\boxed{(x + v\tau) - (x - v\tau) = 2v\tau}$$

with 3-D where $\langle v_x \rangle^2 = \frac{1}{3} \langle v^2 \rangle$

$$\vec{j}^q = \frac{1}{3} n v^2 \tau \, \frac{d\mathcal{E}}{dT} \left(-\nabla T \right)$$

$$\vec{j}^{q} = \frac{1}{3} v^2 \tau \left[\frac{dN\varepsilon}{dT}/V\right](-\nabla T)$$

$$= \frac{1}{3} v^2 \tau \left[\frac{(dE/dT)}{V}\right](-\nabla T)$$

$$\vec{j}^{q} = \frac{1}{3} v^2 \tau c_v (-\nabla T)$$

compare to

$$\vec{j}^{q} = -K(\nabla T)$$

$$\Rightarrow K = \frac{1}{3} v^2 \tau c_v = \frac{1}{3} v \ell c_v \qquad \boxed{\ell = v\tau}$$

$$v^2 = \langle v^2 \rangle$$
= mean square electronic speed

Then if we use

$$\sigma = \frac{ne^2\tau}{m}$$

$$\Rightarrow \frac{K}{\sigma} = \frac{\frac{1}{3} v^2 \cancel{\tau} c_v}{\frac{ne^2 \cancel{\tau}}{m}}$$

$$\frac{K}{\sigma} = \frac{1}{3} \frac{m v^2 c_v}{ne^2}$$

classical ideal gas law: $\Rightarrow$ $c_v = \frac{3}{2} n k_B$

$$\frac{1}{2} m v^2 = \frac{3}{2} k_B T$$

$$\Rightarrow \quad \frac{K}{\sigma} = \frac{1}{3}\, 2\, \frac{\left(\frac{3}{2}k_BT\right)\left(\frac{3}{2}\cancel{n}k_B\right)}{\cancel{n}e^2}$$

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T$$

$$\Rightarrow \text{Lorenze } \# = L = \frac{K}{\sigma T} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2$$

$$= 1.11 \times 10^{-8} \text{ watt-ohm}/K^2$$

<u>but</u> Drude had a mistake; he had found

$$\sigma = \frac{ne^2\tau}{2m}$$

erroneous factor of 2

leading to $\frac{1}{2}$ too small σ

leading to 2x larger L $(\sim 2.22\times10^{-8}$ W-ohm/$K^2)$

in <u>excellent</u> agreement with experiment

Looking back to

$$\frac{K}{\sigma} = \frac{\frac{1}{3}\, C_v\, m\, v^2}{ne^2}$$

actually the true C_v is about 100x smaller than $\frac{3}{2}nk_B$

and the true v^2 is about 100x larger than $\frac{3k_BT}{m}$

cancelling errors "lucky" Drude!

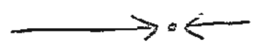
Thermopower

In deriving K, we ignored the effect of the difference in the electrons speed from the hot region and the cold region,

i.e. $\mathcal{E}(T[x - v\tau]) - \mathcal{E}(T[x + v\tau])$

same v

Since hot electron velocity is higher



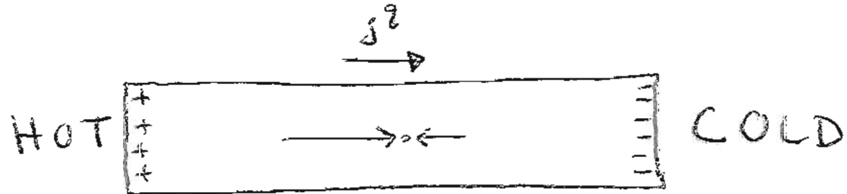
there will be a net electrical current

But only in the case of closed circuit conditions

~~If open circuit~~

In that case there will be a tiny correction of order unity to the K

But if there is Open Circuit



electrons will accumulate on this side if free carriers are electrons

$\Rightarrow \quad \vec{E}$

thermoelectric field

This is the Seebeck effect

Thermoelectric field

$$\vec{E} = Q \nabla T$$

Q = thermopower

<u>mean</u> electronic velocity

$$v_0 = \frac{1}{2}\left[v(x - v\tau) - v(x + v\tau)\right] = -\tau v \frac{dv}{dx}$$

$$= -\tau \frac{d}{dx}\left(\frac{v_x^2}{2}\right)$$

in 3-D $\Rightarrow$ $\vec{V}_Q = -\frac{\tau}{6}\frac{d}{dx}(v^2)$

$$= -\frac{\tau}{6}\frac{dv^2}{dT}\nabla T$$

$$\vec{V}_E = -\frac{e\vec{E}}{m}\tau$$

$$\vec{V}_Q + \vec{V}_E = 0$$

$$\vec{E} = -\frac{m}{e}\left(+\frac{1}{6}\right)\frac{dv^2}{dT}\nabla T$$

$$= \frac{-1}{3e}\frac{d}{dT}\left(\frac{mv^2}{2}\right) = -\frac{c_v}{3ne}$$

with $c_v = \frac{3}{2} n k_B$ $\Rightarrow$ $Q = \frac{-k_B}{2e} = -0.43 \times 10^{-4}$ V/K

<u>But</u> experimentally observed Q's are ~ <u>μV/K</u> !

same factor of 100 error