Density of levels in k-space:

$$\rho_k = \frac{V}{8\pi^3}$$

Consider an N-electron metal in its ground state.
The lowest electronic level is:

$$\vec{k} = 0 \quad \Big\} \text{ lowest-energy 1-electron level}$$

in level $\vec{k} = 0$ we can put 2 electrons (spin $= \pm \frac{\hbar}{2}$)

energy $\propto k^2 \Rightarrow$ levels will fill up forming a sphere

called the "Fermi sphere"

with radius k_F } Fermi wave-vector

and volume $\frac{4}{3}\pi k_F^3 = \Omega$

Number of 1-electron levels $= \Omega \rho_k$

$$= \frac{4}{3}\pi k_F^3 \left(\frac{V}{8\pi^3}\right)$$

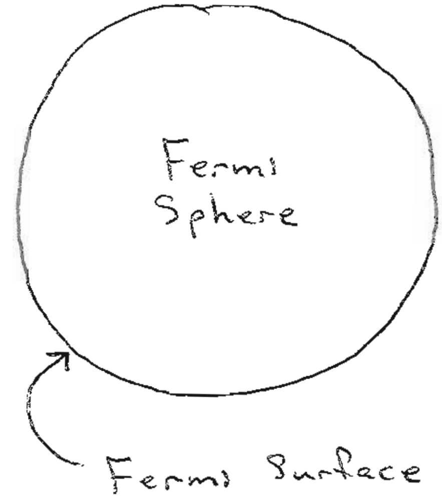
$$\# \text{ 1-electron levels} = \frac{k_F^3}{6\pi^2} V$$

Number of electrons $= 2\Omega\rho_k$

$$N = \frac{k_F^3}{3\pi^2} V$$

$$n = \frac{N}{V} = \frac{k_F^3}{3\pi^2} \quad \Big\} \text{ electron density}$$

Fermi Surface is the boundary (in k-space) of the occupied 1-electron levels



Fermi momentum

$$\vec{p}_F = \hbar \vec{k}_F$$

Fermi energy

$$\mathcal{E}_F = \frac{\hbar^2 k_F^2}{2m}$$

Fermi velocity

$$\vec{v}_F = \frac{\vec{p}_F}{m} = \frac{\hbar \vec{k}_F}{m}$$

} analogous to the thermal velocity in a classical gas $v = \left(\frac{3kT}{m}\right)^{1/2}$

All Fermi quantities are related to the $\underbrace{\text{conduction electron density}}_{n}$,

$$k_F = \left(3\pi^2 n\right)^{1/3} \qquad \text{but} \quad n = \frac{3}{4\pi r_s^3}$$

$$k_F = \left(\frac{9\pi^2}{4\pi r_s^3}\right)^{1/3} = \frac{(9\pi/4)^{1/3} \frac{1}{a_0}}{r_s \frac{1}{a_0}} \qquad a_0 = .529 \text{ Å}$$

$$\boxed{k_F = \frac{3.63}{(r_s/a_0)} \text{ Å}^{-1}}$$

with $r_s \sim 2\text{-}5\, a_0 \Rightarrow k_F \sim 1 \text{ Å}^{-1}$

$$\lambda_F = \frac{2\pi}{k_F} \sim 6 \text{ Å}$$

$$V_F = \left(\frac{\hbar}{m}\right) k_F = \frac{\hbar}{m} \frac{(3.63)}{(r_s/a_0)}$$

$$= \frac{4.20}{r_s/a_0} \times 10^8 \text{ cm/s} \Big\} \sim 1\% \, c$$

This is just the ground state (at $T=0$).

$$\boxed{\text{Compare to classical gas where } v \propto T^{1/2} \Rightarrow v = 0 \text{ at } T = 0}$$

$$\mathcal{E}_F = \frac{\hbar^2}{2m} k_F^2$$

$$a_0 = \left(\frac{me^2}{\hbar^2}\right)^{-1}$$

$$a_0 = \frac{\hbar^2}{me^2}$$

$$= \frac{\hbar^2 e^2}{2me^2} \left[\frac{3.63 \text{Å}^{-1}}{(r_s/a_0)}\right]^2$$

$$= \frac{e^2 a_0}{2} \frac{(3.63 \text{Å}^{-1})^2}{(r_s/a_0)^2}$$

$$= \left(\frac{e^2}{2a_0}\right)(a_0^2)(3.63 \text{Å}^{-1})^2 \frac{1}{(r_s/a_0)^2}$$

$$= (13.6 \text{ eV})(.529 \text{Å})^2 (3.63 \text{Å}^{-1})^2 \frac{1}{(r_s/a_0)^2}$$

$$\boxed{\mathcal{E}_F = \frac{50.1}{(r_s/a_0)^2} \text{ eV}}$$

For $r_s/a_0 \sim 2-5 \Rightarrow \mathcal{E}_F \sim 2-12.5$ eV

Ground State energy

$$E = 2 \sum_{k<k_F} \frac{\hbar^2}{2m} k^2$$

$$= 2 \sum_k F(\vec{k})$$

$$\sum F(\vec{k}) = \frac{V}{8\pi^3} \sum_k F(\vec{k}) \, \Delta\vec{k}$$

$$\Rightarrow \lim_{V \to \infty} \frac{1}{V} \sum F(\vec{k}) = \int \frac{d\vec{k}}{8\pi^3} F(\vec{k})$$

$$\frac{E}{V} = \frac{2}{V} \sum_k \frac{\hbar^2 k^2}{2m}$$

$$= \frac{2}{8\pi^3} \int_k d\vec{k} \, \frac{\hbar^2 k^2}{2m}$$

$$= \frac{1}{4\pi^3} \int_{k=0}^{k=k_F} 4\pi k^2 dk \, \frac{\hbar^2 k^2}{2m}$$

$$\boxed{\frac{E}{V} = \frac{1}{\pi^2} \frac{\hbar^2 k_F^5}{10m}}$$

$$\frac{E}{N} = \frac{E}{V}\left(\frac{V}{N}\right) =$$

$$n = \frac{N}{V} = k_F^3 / 3\pi^2$$

$$\frac{E}{N} = \frac{1}{\cancel{\pi^2}} \frac{\hbar^2 k_F^5}{10m} \left(\frac{3\cancel{\pi^2}}{k_F^3} \right)$$

$$= \frac{3}{5} \frac{\hbar^2 k_F^2}{2m}$$

$$\boxed{\frac{E}{N} = \frac{3}{5} \varepsilon_F}$$

$$\varepsilon_F = k_B T_F$$

$$T_F = \frac{\varepsilon_F}{k_B} \sim 2 - 12 \times 10^4 \text{ K} \Big\} \text{ Fermi Temperature}$$

Bulk Modulus

with

$$B = \frac{1}{K} = -V \frac{\partial P}{\partial V}$$

$$\boxed{P = \frac{2}{3} \frac{E}{V}} \quad \text{from} \quad \frac{E}{N} \frac{N}{V} = \frac{3}{5} \varepsilon_F n$$

$$n = \frac{k_F^3}{3\pi^2} \qquad E = \frac{3}{5} \varepsilon_F n V$$

$$= \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m}$$

$$P = -\left(\frac{\partial E}{\partial V}\right)_N$$

$$= \frac{3}{5} N \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3}$$

$$[E] = \left[\frac{3}{5} N \frac{\hbar^2}{2m} (3\pi^2)^{2/3} N^{2/3} V^{-2/3}\right]$$

$$= \frac{\frac{2}{3}[E]}{V} = \frac{2}{3} \frac{E}{V}$$

$$B = -V \frac{\partial P}{\partial V} \qquad P \propto V^{-5/3}$$

$$\Rightarrow B = \frac{5}{3} V \left(\frac{P}{V} \right) = \frac{5}{3} P$$

$$B = \frac{5}{3} \left(\frac{2}{3} \frac{E}{V} \right)$$

$$B = \frac{10}{9} \frac{E}{V}$$

$$= \frac{10}{9} \left[\frac{3}{5} \varepsilon_F n \right]$$

$$B = \frac{2}{3} n \varepsilon_F$$

$$B \sim 2 - 200 \times 10^{10} \text{ dynes/cm}^2$$

reasonable agreement with some cases of measured values

$\Rightarrow$ $\boxed{\text{electron gas pressure a factor in resistance of metal to compression}}$