Homework #1: due Monday, Sept. 11 60 pts total

(30) 1. A&M, Prob. 1.1 Poisson Distribution

(20) 2. A&M Prob. 1.2 Joule Heating

(10) 3. derive equation 1.8 in the text of A&M.

Homework #1 Key

Problem 1 : AM 1.1 : Poisson Distn (30 pts)

(10 pts) a) Prob of no collision in prec dt_1 is

$$1 - \frac{dt}{\tau} \qquad \ldots \, | dt_3 | dt_2 | dt_1 |$$

Prob. of no col. in prec. dt_2 is

Total Prob. of no coll. is

$$P_T = \left(1 - \frac{dt_1}{\tau}\right)\left(1 - \frac{dt_2}{\tau}\right)\left(\ldots\ldots\left(1 - \frac{dt_m}{\tau}\right)\right.$$

$$P_T = \left(1 - \frac{dt}{\tau}\right)^m \qquad \text{w/} \quad dt = \lim_{m \to \infty} \frac{t}{m}$$

use binomial expansion

$$\to P_T = 1 + \frac{m!}{(m-1)!}\left(-\frac{dt}{\tau}\right)^1 + \frac{m!}{(m-2)!\,2!}\left(-\frac{dt}{\tau}\right)^2 + \ldots$$

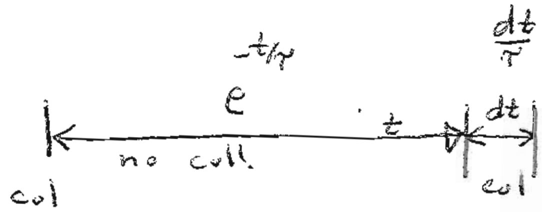
$$= 1 + m\left(-\frac{dt}{\tau}\right) + \frac{m(m-1)}{2}\frac{dt^2}{\tau^2} + \ldots$$

$$= \lim_{m \to \infty}\left[1 + \left(-\frac{t}{\tau}\right) \quad \frac{m(m-1)}{2m^2}\left(\frac{t^2}{\tau^2}\right) + \ldots\right]$$

$$= 1 + \left(-\frac{t}{\tau}\right) + \frac{1}{2}\left(-\frac{t}{\tau}\right)^2 + \ldots$$

$$\boxed{P_T = e^{-\frac{t}{\tau}}}$$

b)
(5 pts)

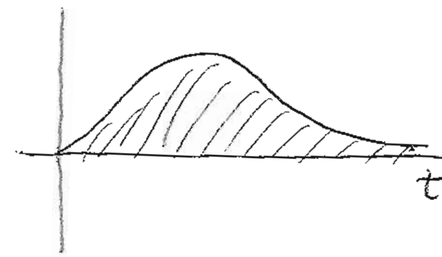


$$\text{Total} = \left(e^{-t/\tau}\right)\left(\frac{dt}{\tau}\right)$$

c)
(5 pts)

$$\text{Prob of no col.} = e^{-t/\tau}$$

$$\langle t \rangle = \frac{\int_0^\infty (t)\, e^{-t/\tau}\, dt}{\int_0^\infty e^{-t/\tau}\, dt}$$



$$\int_0^\infty t\, e^{-t/\tau}\, dt = -\tau\, t\, e^{-t/\tau}\Big|_0^\infty - (-\tau)\int_0^\infty e^{-t/\tau}\, dt$$

$$= \tau\left[-\tau\, e^{-t/\tau}\Big|_0^\infty\right]$$

$$= +\tau^2$$

$$\int_0^\infty e^{-t/\tau}\, dt = -\tau\, e^{-t/\tau}\Big|_0^\infty = \tau$$

$$\Rightarrow \boxed{\langle t \rangle = \frac{\tau^2}{\tau} = \tau}$$

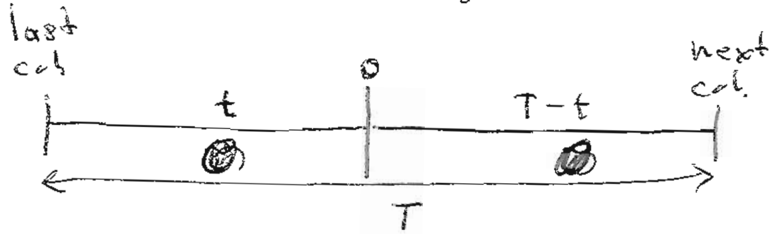
$\Rightarrow$ time back to previous $= \tau$
$+$ time up to next $= \tau$ $\quad\Rightarrow$ interval $= 2\tau$

d) (5 pts)

$$\langle t \rangle = \frac{\int_0^\infty (t)\left[e^{-t/\tau}\,\frac{dt}{\tau}\right]}{\left\{\int_0^\infty e^{-t/\tau}\,\frac{dt}{\tau}\right\} = 1} = \tau$$

successive collisions
mean time between
for a single particle

(e) (5 pts)



last - next = T

using results of (b), calculate $\langle T \rangle$

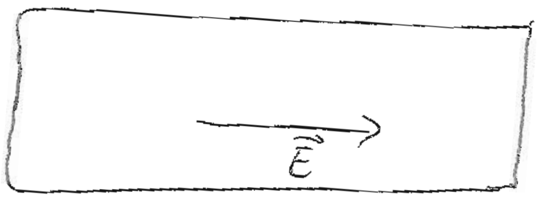
$$P_T = e^{-t/\tau}\, e^{-(T-t)/\tau}\, \frac{dT}{\tau} = e^{-T/\tau}\, \frac{dT}{\tau}$$

$$\langle T \rangle = \frac{\int_0^\infty T\left[e^{-T/\tau}\,\frac{dT}{\tau}\right]}{\left\{\int_0^\infty e^{-T/\tau}\,\frac{dT}{\tau}\right\} = 1} = \tau$$

This implies that
the average of
successive collisions
is τ also

HW #1

Problem 2 : Joule Heating : AM 1.2 (20 pts) Key



(a) Show that energy lost in 2nd col is on average $(eE\tau)^2/2m$ (10 pts)

$$\Delta U = \int_0^t P\,dt = \int_0^t \vec{F}\cdot\vec{v}\,dt$$

$$= \int_0^t (-e\vec{E})\cdot\left(\vec{v}_0 - \frac{e\vec{E}}{m}t\right)dt$$

$$= -e\vec{E}\cdot\vec{v}_0\,t + \frac{e^2E^2}{m}\int_0^t t\,dt$$

$$\Delta U = -e\vec{E}\cdot\vec{v}_0\,t + \frac{e^2E^2t^2}{2m}$$

but average over all emerging directions of $\vec{v}_0$

$\langle \vec{v}_0 \rangle = 0$

$$\Rightarrow \boxed{\Delta U = \frac{e^2E^2t^2}{2m}}$$

(b) P of col. between t and $t+dt$ $= \frac{1}{\tau} e^{-t/\tau} dt$

10 pts

$$\langle U \rangle = \int_0^\infty u P = \int_0^\infty \frac{e^2 E^2 t^2}{2m} \frac{1}{\tau} e^{-t/\tau} dt$$

$$\int_0^\infty x^n e^{-x} dx = n!$$

$$= \frac{\tau^2 e^2 E^2 \tau}{2m\tau} \left[\int_0^\infty \frac{t^2}{\tau^2} e^{-t/\tau} \frac{dt}{\tau} \right] = \frac{\tau^2 e^2 E^2}{2m} [2!]$$

$$\langle U \rangle = \frac{e^2 E^2 \tau^2}{m}$$

average loss / cm^3 / s

$$= \frac{n}{\tau} \langle u \rangle = \frac{n e^2 E^2 \tau}{m} = \sigma E^2$$

$$\sigma = \frac{n e^2 \tau}{m}$$

Power loss in wire of cross-sec A and length L

$$P_T = [\sigma E^2][AL] = \frac{[\sigma E]^2 A^2 L}{\sigma A} = j^2 A^2 \frac{L}{\sigma A}$$

$$P_{Tot} = I^2 \frac{\rho L}{A} = I^2 R$$

Homework #1

Key

Problem #3. (10 pts)

10 pts Show that $\tau = \dfrac{m}{\rho n e^2}$

leads to $\tau = \left(\dfrac{0.22}{\rho_\mu}\right)\left(\dfrac{r_s}{a_0}\right)^3 \times 10^{-14}$ sec

ρ_μ = resistivity in μ ohm-cm

---

use $a_0 = \dfrac{\hbar^2}{me^2}$ and $\dfrac{1}{n} = \dfrac{4\pi r_s^3}{3}$

$$\tau = \frac{\hbar^2}{a_0 e^2}\,\frac{1}{\rho}\,\frac{4\pi r_s^3}{3}\,\frac{1}{e^2}$$

$$\tau = \frac{a_0^2 \hbar^2 (4\pi)}{e^4 \rho\, 3}\left(\frac{r_s}{a_0}\right)^3$$

μ-ohm-cm $= \frac{1}{9} \times 10^{-17}$ statohm-cm

$$= \frac{(.529\times10^{-8}\,\text{cm})^2 (1.05459\times10^{-27}\,\text{erg-sec})^2 (4\pi)}{(4.80324\times10^{-10}\,\text{esu})^4\, 3 \left[\frac{1}{9}\times10^{-17}\right][\mu\text{ohm-cm}]}\left(\frac{r_s}{a_0}\right)^3$$

$$\tau = \left(\frac{0.22}{\rho_\mu}\right)\left(\frac{r_s}{a_0}\right)^3 \times 10^{-14}\ \text{sec}$$