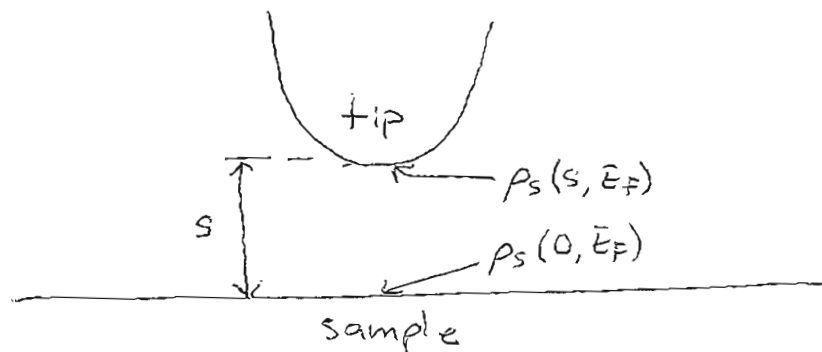


~~XXXXXXXXXX~~

Last time, we discussed how that for a metal-vacuum-metal tunnel junction, the tunnel current I_t will be a function of both the applied tunnel voltage V and the localized density of states of the sample ~~at~~ ^{at} the Fermi level, $\equiv \rho_s(s, E_F)$

$$I \propto V \rho_s(s, E_F)$$

$\rho_s(s, E_F)$ means it's the LDOS at E_F and at the position of the tip



Again, the LDOS of the sample is defined by:

$$\rho_s(z, E) \equiv \frac{1}{E} \sum_{E_n = E - \epsilon}^E |\psi_n(z)|^2$$

Chen, Intro,
to STM, p.6.

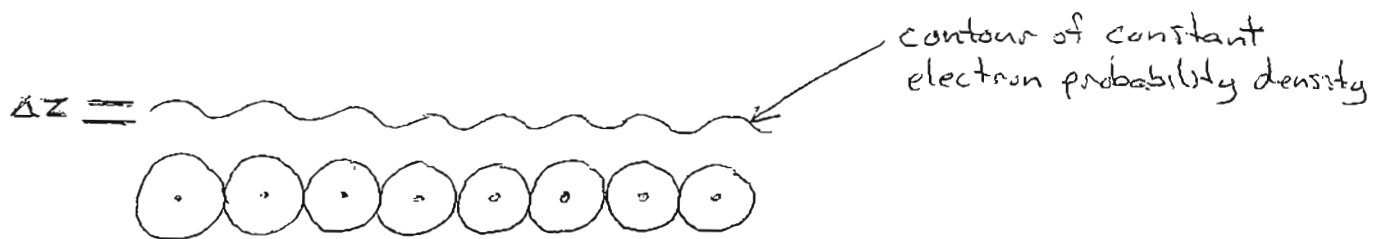
which is related to the square of the wavefunction summed over the states within the energy window $E - \epsilon \rightarrow E$.

In the following, we continue to refer to the book
"Introduction to Scanning Tunneling Microscopy" by C. Julian Chen,
Oxford.

Note in the above expression, that we have not talked
about any "scanning." This is only an expression for two
electrodes separated by a distance s .

When we "scan" the tip over the surface, it is usually
done in "constant current" mode.

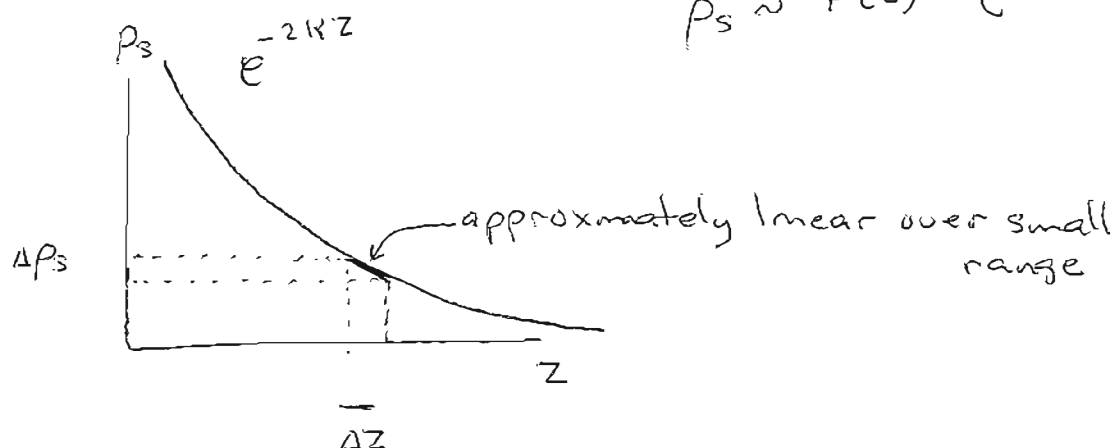
Let's imagine a real surface now, in which the LDOS
varies with position at the "surface." Actually, how do
you define where the "surface" is?



If $I \propto V \rho_s(s, E_F)$,
the the tip needs to follow this contour of constant
electron probability density in order to maintain the
constant current. Then the height "corrugations"
are approximately proportional to the electron probability
density, i.e. ρ_s .

This is because the probability density falls off

$$\rho_s \sim \psi^2(z) \sim e^{-2Kz}$$



Thus in this range, we have:

$$\underline{\rho_s \sim \Delta z}$$

The tip height corrugations relate to the local density of states of the sample surface.

This description is, however, not quite the whole story (by any means). We have not taken into account the tip's density of states. Although if the ρ_t is constant, we find it will be just a constant factor in the equation.

According to Bardeen's theory of tunneling between 2 metal electrodes, the tunneling matrix element is given by a surface integral over a separation surface between the two electrodes.

$$M = \frac{\hbar}{2im} \int_{z=z_0} \left(\chi^* \frac{\partial \psi}{\partial z} - \psi \frac{\partial \chi^*}{\partial z} \right) dS$$

- Chen, p. 22

ψ and χ are wavefunctions of the two electrodes.

Fermi's Golden Rule gives the probability w of an electron in state ψ at energy \bar{E}_ψ to tunnel to a state χ at energy \bar{E}_χ :

$$w = \frac{2\pi}{\hbar} |M|^2 \delta(\bar{E}_\psi - \bar{E}_\chi)$$

Chen, p. 22

↑
only states at same energy
can tunnel into each other

The current can be written as:

$$\frac{4\pi e}{h}$$

$$I = \frac{4\pi e}{h} \int_{-\infty}^{\infty} [f(E_F - eV + E) - f(E_F + E)] \rho_S(E_F - eV + E) \rho_T(E_F + E) \left(\frac{x}{M}\right)^2 de$$

but this may be roughly constant within ΔE

Chen, p. 23

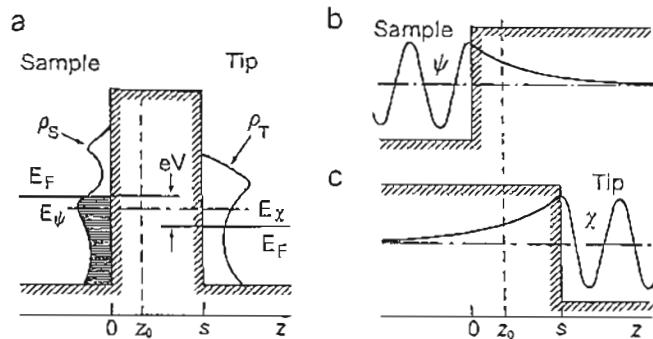
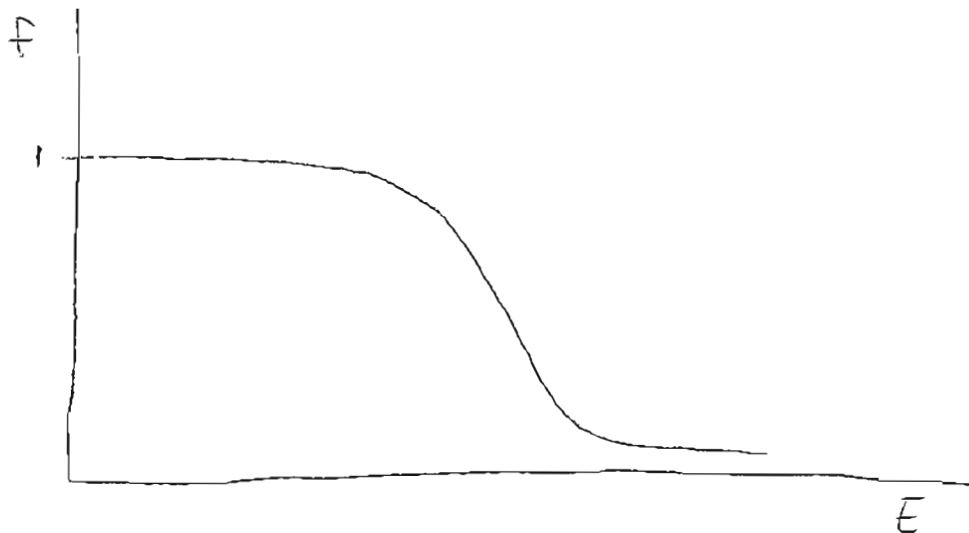


Fig. 1.20. The Bardeen approach to tunneling theory. Instead of solving the Schrödinger equation for the coupled system, a, Bardeen (1960) makes clever use of perturbation theory. Starting with two free subsystems, b and c, the tunneling current is calculated through the overlap of the wavefunctions of free systems using the Fermi golden rule.

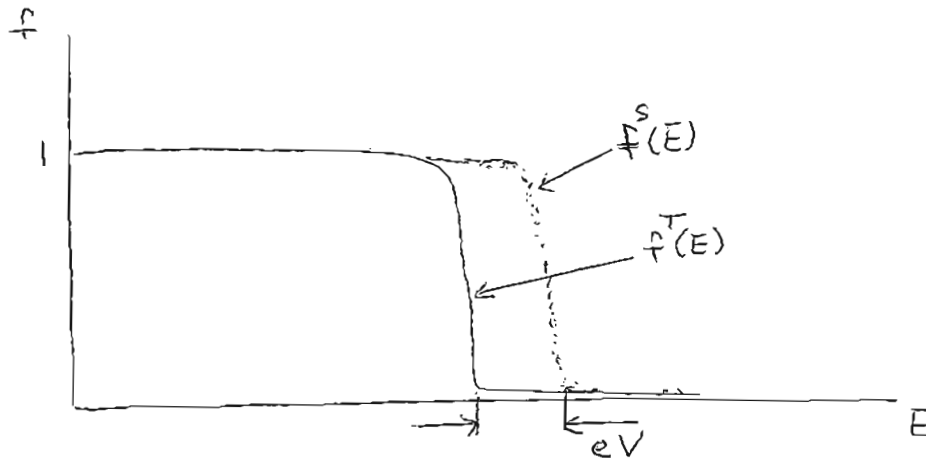
Chen, p. 23

f is the Fermi function:

$$f = \frac{1}{1 + e^{(E - E_F)/kT}}$$



If the energy window (or resolution) is ~~a~~ larger than kT , then the fermi functions can be approximated by step functions:



Then the integral reduces to:

$$I = \frac{4\pi e}{h} \int_0^{eV} P_S(E_F - eV + E) P_T(E_F + E) \overbrace{dE}^{|M|^2}$$

← but this term may be roughly constant within ΔE

Chen, p. 23

$$\Rightarrow I = \frac{C 4\pi e}{h} \int_0^{eV} P_S(E_F - eV + E) P_T(E_F + E) dE$$