Fluctuations in the relaxation of a 2D granular fluid

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Outline

1. Glass Transitions and dynamical heterogeneity
   - Glass transitions
   - Dynamical heterogeneity

2. Relaxation of a fluidized granular system
   - 2D granular system
   - Relaxation

3. Fluctuations in the relaxation
   - Observing spatial fluctuations (Dynamical Heterogeneity)
   - Strength and spatial extent of the spatial fluctuations
At the glass transition, the system “falls out” of equilibrium. Viscosities and relaxation times increase dramatically as a material cools towards $T_g$. 
Dynamical heterogeneity: viscosity - diffusion decoupling

\[ \frac{D\eta}{T} = \text{constant} \]

(Ediger 2000)

(Angell et al. 2000)
Dynamical heterogeneity: non-exponential relaxation

\[ C(t) = C_0 \exp\left(-\left(t/\tau\right)^\beta\right) = \int_0^\infty g(\tau) \exp\left(-t/\tau\right) d\tau = V^{-1} \int d^d r \exp\left(-t/\tau\right) \]

\[ (0 < \beta < 1) \]

(Richert 2002)

(Ediger 2000)
Dynamical heterogeneity: particle tracking

Colloid: confocal microscopy (Courtland and Weeks 2003)

**Figure 4.** Locations of the 10% most mobile particles at three different ages $t_w$. For each picture, mobility was determined by calculating displacements $\Delta r$ over an interval $[t_w, t_w + \Delta T]$, with $\Delta T = 10$ min. Left: $t_w = 10$ min, and $\Delta r > 0.43 \, \mu m$ for the most mobile particles. Middle: $t_w = 55$ min, $\Delta r > 0.34 \, \mu m$. Right: $t_w = 95$ min, $\Delta r > 0.33 \, \mu m$. The data are the same as shown in previous figures, and the choices of $t_w$ correspond to local maxima of $\gamma$ in figure 2(a). The particles are drawn to scale (2.36 $\mu m$ diameter) and the box shown is the entire viewing volume (within a much larger sample chamber).
Dynamical heterogeneity: polarization fluctuations

PVAc: dielectric fluctuations (Vidal Russell & Israeloff, Nature (2000))
- Polymer glass, $T = T_g - 9K$
- Transient appearance of strongly fluctuating region under tip
- Heterogeneity lifetime $\approx$ relaxation time
Granular system and simulations

- 50:50 mixture of hard disks \( r_2/r_1 = 1.43 \).
- \( N_{\text{tot}} = 4 \times 10^6 \) (in most cases).
- Packing fractions \( \phi = 0.60 \cdots 0.805 \).
- Collisions: \( \mathbf{n} \cdot (\mathbf{v}'_i - \mathbf{v}'_j) = -\epsilon \mathbf{n} \cdot (\mathbf{v}_i - \mathbf{v}_j) \).
- Restitution coeff \( \epsilon = 0.7, 0.8, 0.9 \) (inelastic), \( \epsilon = 1.0 \) (elastic).
- Energy restored via random “kicks” to pairs of particles:
  \( m_i \mathbf{v}'_i = m_i \mathbf{v}_i + p_{Dr} R_i \).
Probing relaxation

- Glass $\leftrightarrow$ “random solid”.
- Solid $\leftrightarrow$ “frozen”, no/slow change in time.
- Two-time correlation $C(t, t_w)$:

\[ C(t, t_w) \approx \begin{cases} 
  1 & : \text{unchanged between } t_w \text{ and } t \\
  0 & : \text{correlation lost between } t_w \text{ and } t 
\end{cases} \]

- Spin glass: $S_i = \pm 1 \rightarrow C(t, t_w) \equiv \frac{1}{N} \sum_i S_i(t)S_i(t_w)$.
- Structural glasses / granulars: $\vec{r}_i \rightarrow Q(t) \equiv \frac{1}{N} \sum_i w_i(t)$

\[ w_i(t) \equiv \theta(a - |\vec{r}_i(t) - \vec{r}_i(0)|). \]

Relaxation time $\tau_\alpha \leftrightarrow Q(\tau_\alpha) = 1/e$. 

\[ \begin{array}{c|c}
  Q(t) & 1/\tau_\alpha \\
  \hline
  1 & 10^0 \\
  0.9 & 10^1 \\
  0.8 & 10^2 \\
  0.7 & 10^3 \\
  0.6 & 10^4 \\
  0.5 & 10^5 \\
  0.4 & 10^6 \\
  0.3 & 10^7 \\
  0.2 & 10^8 \\
  0.1 & 10^9 \\
\end{array} \]
Relaxation function and relaxation time

Relaxation function $Q(t)$ for $\phi = 0.60, \cdots, 0.805$

![Graph showing relaxation function $Q(t)$ for different packing fractions.](image)

Relaxation time vs. packing fraction

$\tau_\alpha \propto (\phi_0 - \phi)^{\gamma_\tau}$

$\tau_\alpha \propto \exp[B/(\phi_0 - \phi)]$

(Avila, HC, Vollmayr-Lee & Zippelius, 2015, submitted)
Scaling of the relaxation

No time-density superposition

Two-parameter scaling

\[ Q(t) \approx \left( \frac{t}{\tau_0} \right)^{-\beta} \]

(Avila, HC, Vollmayr-Lee & Zippelius, 2015, submitted)
Dynamical heterogeneity: slow and fast regions

(a) $\phi = 0.60$  
(b) $\phi = 0.78$  
(c) $\phi = 0.805$

Mobility:  
- Slow $< a$  
- Fast $> 3a$

(Avila, HC, Vollmayr-Lee & Zippelius, 2015, submitted)
Dynamical susceptibility

\[ \chi_4(t) = N \left[ \langle Q^2(t) \rangle - \langle Q(t) \rangle^2 \right] \]


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Four point dynamic structure factor

$W(q, t) \equiv \frac{1}{N} \sum_i w_i(t) \exp(iq \cdot r_i(0))$

$S_4(q, t) = N \left[ \langle W(q, t)W(-q, t) \rangle - | \langle W(q, t) \rangle |^2 \right]$

Correlation length $\xi$ and dynamic susceptibility $\chi_4$

\[ S_4(q, t) = \frac{\chi_4(t)}{1 + [q\xi(t)]^2} \]

Dependence on restitution coefficient $\epsilon$

Scaling of correlation volume with correlation length

\[ \chi_4(\tau_\alpha) \propto \xi^{1.58} \]

Summary

- Strong slowdown of relaxation as packing fraction is increased.
- The relaxation function does not satisfy time-density superposition, but it satisfies two-parameter scaling with a simple scaling function.
- Slow and fast particles are found in correlated regions, with size that grows dramatically with packing fraction $\phi$.
- The strength of the fluctuations, probed by $\chi_4(t)$, has a peak whose position and strength increase with $\phi$.
- The 4-point dynamic structure factor for all $\phi$ and $\epsilon$ collapse as a function of $\xi q$.
- Both $\chi_4(\tau_\alpha)$ and $\xi(\tau_\alpha)$ have power law divergences with $\phi$.
- The fluctuations are strongly dependent on the degree of inelasticity.
- The relation between correlated volume ($\propto \chi_4(\tau_\alpha)$) and correlation length ($\xi(\tau_\alpha)$) is a robust, $\epsilon$-independent, power law, with power $p \sim 1.6$ corresponding to a fractal object.
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