Few-Nucleon Calculations without Angular Momentum Decomposition

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Research Objectives and Significance

The objective is to carry out few-nucleon calculations in the intermediate energy regime. In this regime projectile energies usually range from a few hundred MeV to a few GeV. Experimentally this energy regime has been studied experimentally quite intensively in recent decades [1] and is still under investigation at the Cooler Synchrotron COSY (Germany), the Research Center for Nuclear Physics RCNP (Japan) and the new HIRFL-CSR ion accelerator complex at the Institute for Modern Physics IMP (China). At these facilities e.g. proton-deuteron (pd) reactions and breakup processes are experimentally investigated. The theoretical interpretation faced and still faces serious challenges. In the intermediate energy regime pion production channels are open and nuclear resonances play a role. In contrast to the energy regime below pion threshold, where high precision nucleon-nucleon (NN) forces are established, the intermediate energy regime does not yet have forces of comparable quality [2]. Moreover, it an be expected that three-nucleon forces (3NF) will play a more dominant role.

A consistent treatment of intermediate energy reactions requires a Poincaré symmetric quantum theory [3]. In addition, the standard partial wave decomposition, successfully applied below the pion production threshold [4], is no longer an adequate numerical scheme due to the proliferation of the number of partial waves. Thus, the intermediate energy regime is a new territory for few-body calculations, which waits to be explored.

Computational Approach

The discretized Faddeev equation for the scattering of 3 identical particles (neglecting spin and isospin degrees of freedom for our first calculations) is a three dimensional integral equation in 5 variables. These 5 variables are the magnitudes of the the Jacobi (Poincaré-Jacobi) momenta p and q, and angles between the momentum vectors and a fixed projectile momentum $\mathbf{q_0}$. Typical grids for the momentum variables have 50 to 60 points, and for the angular variables are between 20 and 36 points (leading to maximum grid sizes of about 10^8 points). The integral equation is solved iteratively, first the Neumann series is generated by applying the kernel of the equation successively on the start vector, then the series is summed as Padé approximants. A converged result needs typically 15 approximants at lower energies and about 8-10 at higher energies.

For the integration itself, Gaussian quadrature is used. For the kernel of the integral equation, a two-body t-matrix (with the two-nucleon interaction as driving term) is obtained by solving a system of linear equations of the form A*x=b, where A is typically a 2000*2000 matrix. This system is solved for about 80 different vectors b. In case of the Poincaré invariant calculation the boost is generated by a further 1st resolvent type integral equation for each of the off-shell momenta needed in the calculation [5]. Since the momentum region which contributes to a solution of the two-body t-matrix is quite different from the region of importance in a 3-body calculation, the solutions of the two-body amplitude are then mapped onto a momentum grid relevant for the Faddeev equations by repeated application of the Lippmann-Schwinger equation. For treating the deuteron pole, the residue of the t-matrix at the pole has to be extracted with high precision. The moving singularities inherent in any 3-body calculation above the 3-body break-up, are treated by subtraction, and the logarithmic singularities of the subtraction term are integrated by a spline method, i.e. it is semi-analytic. The latter proved to be important for the accuracy and simplicity of the calculation. The interpolations needed to solve the 3-body transition amplitude are based on cubic Hermite splines. The number of required interpolations is typically 10^9.

Accomplishments

At present we accomplished to aspects of the list of challenges: exact Poincaré invariance and calculations using vector variables instead of partial waves. In a series of publications [6] the technique of solving the nonrelativistic momentum-space Faddeev equation without partial waves has been mastered, for bound as well as scattering states (using spinless interactions). The Faddeev equation, based on a Poincaré invariant mass operator has been formulated in detail and solved in first order in the two-body interaction embedded into the three-body Hilbert space. The resulting Faddeev equation has both, kinematical and dynamical differences with respect to the corresponding non-relativistic equation. However the operator form is similar, so that the nonrelativistic codes could be extended in a straightforward fashion.

In [5] the driving term in the relativistic Faddeev equation (first order in the two-body transition operator) has been used to evaluate *pd* elastic as well as breakup scattering. This has now been completed by fully solving the relativistic Faddeev equation using the numerical techniques previously used to solve the nonrelativistic equation. Our calculations converge well up to 2 GeV, indicating the applicability of the formulation of the Faddeev equation based on vector variables for intermediate energies. As example we show in Fig. 1 a calculation for exclusive *pd* breakup at projectile kinetic energy 508 MeV, which is compared to a $^2H(p,2p)n$ experiment [7]. It is very obvious that already at this energy the relativistic effects are quite large and differ by about an order of magnitude from the nonrelativistic results. We noted with surprise that despite our simple NN interaction model our relativistic calculation comes close to the data.

References

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Relevant Publications

1. Relativistic Effects in Exclusive pd Breakup Scattering at Intermediate Energies, T. Lin, Ch. Elster, W.N. Polyzou, W. Glöckle, nucl-th/0710.4056, submitted to Phys. Lett. B.

2. Relativistic three-body scattering in a first order Faddeev formulation, Ch. Elster, T. Lin, W.N. Polyzou, W. Glöckle, nucl-th/0708.3868, to appear in the proceedings of the INPC 2007.

3. First Order Relativistic Three-Body Scattering, T. Lin, Ch. Elster, W.N. Polyzou, W. Glöckle, nucl-th/0702005, Phys. Rev. C**76**, 014010 (2007).

4. Three-Body Elastic and Inelastic Scattering at Intermediate Energies, H. Liu, Ch. Elster, W. Glöckle, nucl-th/0610006, Nucl. Phys. **A 790**, 262c (2007).

5. Three-Body Scattering at Intermediate Energies, H. Liu, Ch. Elster, W. Glöckle, Phys. Rev. C**72**, 054003 (2005).

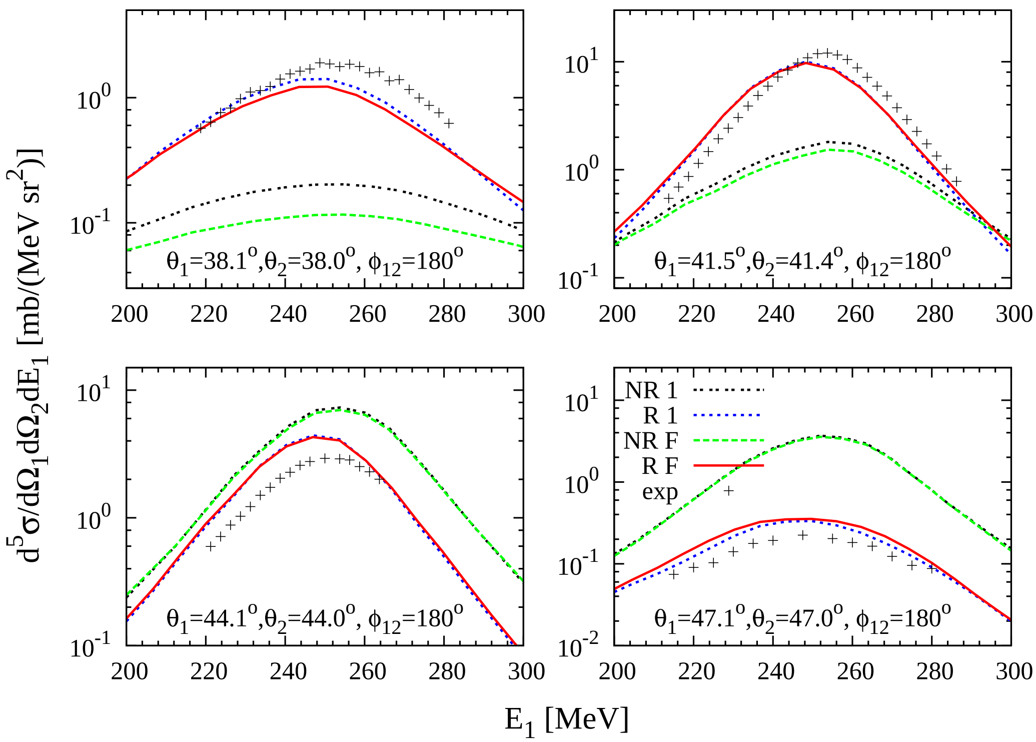

Figure 1: The exclusive differential cross section for the (p,2p) reaction at 508 MeV laboratory projectile energy for different proton angle pairs θ_1-θ_2 symmetric around the beam axis as function of the laboratory kinetic energy of one of the outgoing protons. The solid line (R F) represents the full relativistic solution of the Faddeev equation, while the dotted curve (R 1) indicates the relativistic calculation based on the 1st order in the multiple scattering expansion of the Faddeev amplitude. The corresponding non-relativistic full solution of the Faddeev equation is given by the short-dashed curve (NR F) and its 1st order contribution by the double-dotted curve (NR 1). The data are taken from Ref. [7].