1. Polarizability: Particle in a Box [8 pts]
For a particle of mass $m$ and charge $e$ in its ground state when confined to a box $-a \leq x \leq a$, $-b \leq y \leq b$, and $-c \leq z \leq c$ find the electric polarizability.
The polarizability $\alpha$ is obtained from the shift in energy
\[ \Delta E = -\frac{1}{2} \alpha \mathcal{E}^2 \]
when the particle is placed in an electric field $\mathcal{E}$.

2. Rigid Rotator in Magnetic Field
A system with moment of inertia $I$ has the Hamiltonian
\[ H_0 = \frac{L^2}{2I} \]

(a) (4 p) What are the energies of the lowest and first excited states?
(b) (6 p) A perturbation
\[ H' = g \frac{eB}{Mc} L_x \]
is applied. Find the splitting of the first excited states.

3. Beta Decay of Tritium (6 p)
A tritium atom ($^3\text{H}$) in its ground state $\beta$-decays to form a singly ionized helium atom ($^3\text{He}$). What is the probability that this new atom will be found in its ground state? Assume that both nuclei have infinite mass and that there is no interaction between the beta-decay electron and the rest of the system.

4. Photo-disintegration of the Deuteron (8 p)
Let the deuteron be an s-wave ($l=0$) bound state of a proton and neutron with a binding energy of 2.226 MeV. It is well approximated as a bound state in a square well of depth $V_0 = 36.2$ MeV and width $a = 2.02 \times 10^{-13}$ cm. Using these data, compute the probability
for photo-disintegration of the deuteron. Assume that the incident photon can be approximated by a perturbation

\[
V = \begin{cases} 
  e\vec{A} \cdot \vec{r} \sin \omega t & t > 0 \\
  0 & t < 0 
\end{cases}
\]

where \( \vec{A} \) is a constant vector of magnitude about 1 x 10^3 V/cm. Use whatever other approximations seem reasonable.