1.  
(a) [4 pts]  
The long range part of the nuclear force is determined by the Yukawa potential  
\[ V(r) = \frac{\beta}{r} e^{-\mu r}, \]  
where \( \beta \) is the strength (or coupling constant) and \( \mu \) is given through the pion mass via \( \mu = m_{\pi} \sqrt{\alpha} \). Show that in the Born approximation the differential cross section for the scattering vector \( \vec{q} = \vec{p}' - \vec{p} \) is given by  
\[ \frac{d\sigma}{d\Omega} = \left( \frac{2m\beta}{\hbar^2(q^2 + \mu^2)} \right)^2 \]  
(b) [3 pts]  
Use the result from (a) to determine the properties of the differential cross section in forward direction as well as for the case \( E \rightarrow 0 \).  

(c) [4 pts]  
If you use the Coulomb potential \( V^C = -Ze^2/r \) in the expression for the Born approximation for the scattering amplitude, you obtain a non-existing expression (which one?). One can apply the mathematical trick of solving for a screened Coulomb potential of the form  
\[ V^R(r) = -\frac{Ze^2}{r} e^{-r/R} \]  
and then consider the limit \( R \rightarrow \infty \).  
Derive the scattering amplitude for the screened Coulomb potential and consider the limit \( R \rightarrow \infty \) to obtain the Coulomb Born amplitude and from it the classical Rutherford cross section  
\[ \frac{d\sigma^b}{d\Omega} = \frac{Z^2e^4m^2}{4\mu^2 \sin^2 \frac{\theta}{2}} \]
2. Neutrons of mass $m$ and energy $E$ are incident on a spherically symmetric, square-well, attractive potential of depth $W$ and range $a$, representing the nuclear force between the neutron and a nucleus. If the velocity $v \ll \hbar/ma$, show that

(a) [3 pts]
The scattering is spherically symmetric.

(b) [4 pts]
The s-wave phase shift $\delta_0$ satisfies

$$j \tan(ka + \delta_0) = k \tan ja$$

where

$$k^2 = \frac{2mE}{\hbar^2}, \quad j^2 = \frac{2m(W + E)}{\hbar^2}$$

(c) [4 pts]
The scattering length is given by

$$b = a \left( 1 - \frac{\tan y}{y} \right),$$

where

$$y = \sqrt{2mWa/\hbar}$$

(d) [3 pts]
Calculate the total scattering cross-section as $E$ tends to zero.

(e) [5 pts]
If the value of $y$ for the potential lies in the range $(n - \frac{1}{2})\pi < Y < (n + \frac{1}{2})\pi$, where $n$ is an integer, show that, for a neutron in this potential, there are $n$ bound states (i.e. states of negative energy) with zero orbital angular momentum.

(f) [4 pts]
Show that, if the scattering length is positive, there is at least one bound state.