On the Empirical Optical Potential for Nucleon-Nucleus Scattering

Chapter 4.7 in the textbook ‘Introduction to Nuclear Reactions’ by Bertulani and Danielewicz describes the basic ideas leading to empirical optical potentials for proton (or neutron) nucleus scattering. Let us consider a recent work on ‘Isospin dependent global nucleon-nucleus optical model at intermediate energies’ by S.P. Weppner, R.B. Penney, G.W. Diffendale and G. Vittorini, Phys. Rev. C 80, 034608 (2009), and compare the textbook version to a current realistic optical model potential.

1. (3 p) State briefly the goals of the work by Weppner et al. and establish in which regime their optical potential fit should be valid.

2. (10 p) Take the general expression for $U(r)$ given in Eq. (4.81) of the textbook and compare this term by term to the terms used by Weppner et al. Pay specific attention to how the authors choose their functions and count the number of parameters they have for each term of Eq. (4.81).

3. (6 p) Describe and comment on terms the WP potential has in addition to the ones given in the textbook. Do the authors justify extra terms? If not, can you think of a justification why those terms would make physical sense and are beneficial to include.

4. (5 p) In order to make simple estimates of nuclei, a heavy nucleus can be represented by a spherical spherical square well potential

$$V(r) = \begin{cases} -V_0, & r < R \\ 0, & r > R \end{cases}$$

where $V_0$ is a positive constant. Calculate the differential cross section for scattering of a nucleon from this potential in first Born approximation. Proceed by

(a) Showing that for spherically symmetric potential the scattering amplitude $f_k(\theta)$ is the Fourier transform of the potential with respect to the variable $q = 2k \sin \frac{\theta}{2}$.

(b) Explicitly calculating the scattering amplitude in Born approximation for the potential given above.
5. (3 p) How can the result you obtained above be used to estimate the quantity $R$?

6. (12 p) Estimate the size of $^{56}$Fe. Proceed by
   
   (a) Go to the authors web-site given in Ref. [11] and obtain the differential cross section for neutron scattering off $^{56}$Fe at an energy larger than 100 MeV. Plot the cross section as function of c.m. angle.

   (b) Though higher order rescattering contributions prevent the cross section to show a zero, you can use the first diffraction minimum to estimate the size of the target nucleus,

   (c) Compare your own estimate with rough estimate the the size of a given nucleus is proportional to $r_0 A^{1/3}$, where $A$ is the total number of nucleons (neutrons and protons) in a nucleus and $r_0 = 1.25$ fm.

7. (4 p) Explain the meaning of isoscalar and isovector, and illustrate your explanation using the paper.

8. (3 p) The WP fit looks worst for the two carbon isotopes. Do you have a suggestion what the reason could be? Provide any physical insight you have or find.