Phys. 735: Homework I

due September 13, 2006

1. Schrödinger Equation: Translational Invariance

(a) *(6 pts)*
Show that the Schrödinger equation for a free particle

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi \tag{1}$$

is form invariant under a translation

$$\begin{aligned} x &\rightarrow x' = x - vt \\ y &\rightarrow y' = y \\ z &\rightarrow z' = z. \end{aligned} \tag{2}$$

Explicitly, show that if the wave function in the new coordinate system is given by $\Psi'(x', y', z', t)$, then it satisfies

$$i\hbar \frac{\partial \Psi'}{\partial t} = -\frac{\hbar^2}{2m}\nabla'^2 \Psi'. \tag{3}$$

Hint: The wave function $\Psi'(x', y', z', t)$ is a scalar and transforms as scalar with an additional phase transformation.

(b) *(6 pts)*
Apply your result to a plane wave $\Psi(\mathbf{x}, t) = e^{i[\omega t - \mathbf{k}\cdot\mathbf{x}]}$ and discuss the result you obtain as far as energy and momentum are concerned.

2. Free motion in an accelerated frame *(10 pts)*
Show that if in the Schrödinger equation for a free particle moving in x-direction

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} \tag{4}$$

one goes to an accelerated frame given by

$$x \rightarrow x' = x + \frac{1}{2}at^2 \tag{5}$$

the wave function $\Psi'(x', t)$ in the new coordinate system satisfies a Schrödinger equation of the form

$$i\hbar \frac{\partial \Psi'(x', t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi'(x', t)}{\partial x'^2} + V(x')\Psi'(x', t), \tag{6}$$

where $V(x') = -max'$ is the potential for a constant force that produces a uniform acceleration a.

Hint: In transforming the wave function in to the accelerated frame allow for a phase factor so that the wave function in the stationary frame $\Psi(x, t)$ and the wave function in the accelerated frame are related by

$$\Psi(x, t) = e^{i\, f(x', t)} \Psi'(x, t). \tag{7}$$

and determine f.