

## Phys. 735: Homework V

due October 30, 2006

### 1. (8 pts)

The normalized, coordinate-space plane wave solution of the Dirac equation is given as

$$\Phi_s^{(\pm)}(x) = e^{\pm i p_\mu x^\mu} u_s^{(\pm)}(\mathbf{p}), \quad (1)$$

where the space-time dependence is in the exponential, the 4D spin dependence in the spinor, and  $\mathbf{p}$  is *not* an operator.

When this plane wave solution is substituted into the coordinate-space Dirac equation one obtains the *momentum-space Dirac equation* for the free particle spinors  $u_s^{(\pm)}(\mathbf{p})$ :

$$(\gamma^\mu p_\mu \mp m) u_s^{(\pm)}(\mathbf{p}) = 0 \quad (2)$$

(a) Prove the relation (2) starting from (1) for  $u_s^{(+)}(\mathbf{p})$  and  $u_s^{(-)}(\mathbf{p})$ .

(b) The adjoint spinor  $\bar{u}_s^{(\pm)}(\mathbf{p}) = u_s^{(\pm)\dagger}(\mathbf{p})\gamma^0$  satisfies the transpose Dirac equation

$$\bar{u}_s^{(\pm)}(\mathbf{p})(\gamma^\mu p_\mu \mp m) = 0 \quad (3)$$

Prove the above equation (3) for both cases.

### 2. (10 pts)

When manipulating solutions of the Dirac equation, it is often useful to combine groups of them into a simple operator. To do this, define the projection operators

$$\Lambda_\pm(p) = \frac{\pm \gamma^\mu p_\mu + m}{2m} \quad (4)$$

(a) Show that these operators are projection operators, i.e. fulfill

$$\begin{aligned} \Lambda_\pm^2(p) &= \Lambda_\pm(p) \\ \Lambda_- \Lambda_+ &= \Lambda_+ \Lambda_- = 0 \\ \Lambda_+ + \Lambda_- &= \mathbf{1} \end{aligned} \quad (5)$$

(b) Show that these operators have the desired effect when applied on the Dirac spinors:

$$\begin{aligned} \Lambda_\pm(p) u_s^{(\pm)}(\mathbf{p}) &= u_s^{(\pm)}(\mathbf{p}) \\ \Lambda_\mp(p) u_s^{(\pm)}(\mathbf{p}) &= 0 \end{aligned} \quad (6)$$

**3.** (8 pts)

In relating field theoretic amplitudes to nonrelativistic potentials, it will be necessary to know nonrelativistic limits of the bilinear covariants. Determine the leading order terms in  $p/m$  for

$$\begin{aligned} \text{(a)} \quad & \bar{u}_s(\mathbf{p}')\gamma^0 u_s(\mathbf{p}) \\ \text{(b)} \quad & \bar{u}_s(\mathbf{p} + \mathbf{q})\gamma^5 u_s(\mathbf{p}) \end{aligned} \tag{7}$$