Treatment of Higher Spin Fields

VLADIMIR PASCALUTSA

Department of Physics and Astronomy

Ohio University, Athens, Ohio

Motivation

Low-energy QCD in terms of hadronic DOF

$$\mathcal{L}_{eff} = \partial_\mu \pi^* \partial^\mu \pi - m_\pi^2 \pi^* \pi + \overline{N}(i\partial\!\!\!/ - m_N)N$$
$$+(g_A/f_\pi)\overline{N}\gamma_\mu\gamma_5 N \partial^\mu \pi + \ldots$$

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&\quad + \mathcal{L}_\Delta + \mathcal{L}_{\pi N \Delta} + \mathcal{L}_{\pi\Delta\Delta} + \ldots
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Motivation

Hadron	s	Field	Lorentz property
Pion:	0	$\pi(x)$	pseudo-scalar
Nucleon:	$1/2$	$N^{(\alpha)}(x)$	spinor
Rho-meson:	1	$\rho^{\mu}(x)$	vector
Delta:	$3/2$	$\psi^{\mu(\alpha)}(x)$	vector-spinor

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In general,

Spin-s meson: $h^{\mu_1 \cdots \mu_s}(x)$

Spin-s baryon: $\psi^{\mu_1 \cdots \mu_{s-1/2}}(x)$

Spin-DOF counting

Difficulty: $h^{\mu_1 \cdots \mu_s}(x)$, $\psi^{\mu_1 \cdots \mu_{s-1/2}}(x)$ have more components than is needed to describe particles with

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E.g., massless $s = 1$:

$$\mathcal{L} = A_\mu \partial^2 A^\mu$$

leads to 4 spin DOF — does not work.

Local (gauge) symmetries reduce the # DOF. In general, invariance under $h \to h + D^{(d)}\phi^{(n)}(x)$, where d is the order of the diff. op. D, and ϕ are n independent parameters, gets rid of $n\,(d+1)$ DOF.

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With

$$\delta\mathcal{L} = 0, \quad \text{under} \quad \delta h^{\mu_1 \cdots \mu_s}(x) = \partial^{\{\mu_1} \phi^{\mu_2 \cdots \mu_s\}}$$

it is generally possible to reduce to only 2 DOF.

Then the mass term is introduced such this HS symmetry is broken to raise the DOF number to 2s+1.

EM field, A_μ: $\mathcal{L} = -(1/4)F^2$ is invariant under $\delta A_\mu = \partial_\mu \phi$, so that # DOF =4-2=2. Mass term, $m^2 A^2$, provides # DOF =3.

N YM fields, A_μ^a ($4N$ components). Invariant under $\delta A_\mu^a = (\delta^{ab}\partial_\mu + g f^{abc} A_c)\phi^b$. # DOF =4N-2N=2N.

Free spin-3/2 field

Spin-3/2 particle is described by a 16-component ψ_μ. The Lagrangian,

$$\mathcal{L} = \bar{\psi}_\mu \gamma^{\mu\nu\alpha} \partial_\alpha \psi_\nu ,$$

with $\gamma^{\mu\nu\alpha} = (\gamma^\mu\gamma^\nu\gamma^\alpha - \gamma^\alpha\gamma^\nu\gamma^\mu)/2$, describes free massless spin-3/2 particle. Due to the gauge symmetry:

$$\psi_\mu \to \psi_\mu + \delta\psi_\mu, \;\; \delta\psi_\mu = \partial_\mu \epsilon ,$$

$\epsilon(x)$ is a spinor field, there are only 2 independent DOF.

Massive free spin-3/2 field

The mass term is introduced such that # DoF = 2s+1=4.

One finds

$$\mathcal{L} = \bar{\psi}_\mu \gamma^{\mu\nu\alpha} \partial_\alpha \psi_\nu - m \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu$$

with $\gamma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]/2.$

Similarly for higher s. Free formulation was done in 70's.

Interactions are problematic (e.g., Johnson-Sudarshan, Velo-Zwanziger problems)

A consistency condition

Interactions must be consistent with the DOF counting. Warranted if interactions are gauge-symmetric (both for $m = 0$ and not), $\delta\mathcal{L}_I = 0$.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_m + \mathcal{L}_I$$

$$\delta\mathcal{L} = \delta\mathcal{L}_m \sim m \,,$$

hence only the mass term is allowed to raise the number of DOF.

Ref: V. P., Phys Rev D 58 (1998) 096002.

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Given that, gauge invariance infers a transversality condition on the a Green’s function involving a HS leg:

$$p_{\{\mu_1} \Gamma^{\mu_1 \dots \mu_s\}}(p, \cdots) = 0.$$

πNN* and γNN* couplings

Couple to explicitly invariant quantities, e.g., in s=3/2 case

$$G_{\mu\nu} = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu, \quad \tilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} G_{\mu\nu}$$

$\pi N \Delta$ and $\gamma N \Delta$ couplings:

$$\mathcal{L}_{\pi N \Delta} = \frac{f_{\pi N \Delta}}{m_\pi m_\Delta} \bar{\Psi}\, \gamma_5\, \gamma_\mu\, \tilde{G}^{\mu\nu}\, \partial_\nu\, \phi + \text{H.c.}$$

$$\mathcal{L}_{\gamma N \Delta} = \frac{3e}{2m_N(m_N + m_\Delta)} \left[g_M\, \bar{\Psi} T_3 G_{\mu\nu} \tilde{F}^{\mu\nu} + g_E\, \bar{\Psi} T_3 \gamma_5 G_{\mu\nu} F^{\mu\nu} \right]$$

Ref: V. P. and D. Phillips, nucl-th/0305xxx.

Decoupling of spin-1/2 sector

The spin-3/2 (Rarita-Schwinger) propagator

$$\begin{aligned} S_{\mu\nu}(p) &= \frac{1}{\not{p} - m} P^{(3/2)}_{\mu\nu} - \frac{2}{3m^2}(\not{p} + m) P^{(1/2)}_{22,\mu\nu} \\ &+ \frac{1}{\sqrt{3}m}\left(P^{(1/2)}_{12,\mu\nu} + P^{(1/2)}_{21,\mu\nu}\right) \end{aligned} \tag{1}$$

where $P^{(J)}_{mn,\mu\nu}$ are the spin-projection operators.

$$P^{(3/2)}_{\mu\nu}(p) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}(\not{p}\gamma_\mu p_\nu + p_\mu\gamma_\nu\not{p})$$

projects onto the pure spin-3/2 states

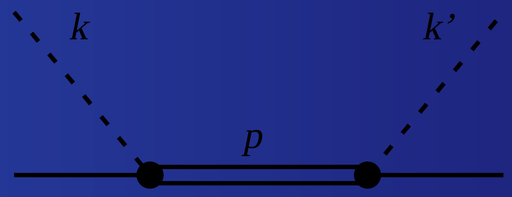
$$
\begin{aligned}
P^{(1/2)}_{22,\mu\nu} &= p_\mu p_\nu / p^2 \ , \\
P^{(1/2)}_{12,\mu\nu} &= p^\varrho p_\nu \sigma_{\mu\varrho} / (\sqrt{3}\, p^2) \ , \\
P^{(1/2)}_{21,\mu\nu} &= p_\mu p^\varrho \sigma_{\varrho\nu} / (\sqrt{3}\, p^2) \ ,
\end{aligned}
$$

project onto the spin-1/2 sector, which decouples due to:

$$
p_\mu \Gamma^\mu(p,k) = 0, \quad => \Gamma^\mu \, P^{(1/2)}_{\mu\nu} \Gamma^\nu = 0.
$$

A typical amplitude:

$$\Gamma^{\mu}(p,k')\,S_{\mu\nu}(p)\,\Gamma^{\nu}(p,k) =$$



$$= \frac{(f_{\pi N\Delta}/m_\pi)^2}{\not p - m}\,\frac{p^2}{m^2}\,P^{(3/2)}_{\mu\nu}(p)\,k'^{\mu}k^{\nu}$$

$P^{(3/2)}(p)$ the spin-3/2 projection operator ($p^2\,P^{(3/2)}$ is local).

Generalized to other spins. Due to

$$p_{\{\mu_1} \Gamma^{\mu_1 \cdots \mu_{s-1/2}\}} = 0,$$

full propagator can be replaced by the highest spin term,

etc.

One can easily write down an expression for a hadron-exchange amplitude of arbitrary spin.

$\pi N \to \pi N$ (*Ref: V. P. and J. A. Tjon, PRC (2001)*), $\gamma N \to \pi N$, $\gamma N \to \gamma N$ (*to appear somewhere eventually*).

4. Equivalence theorem

"Conventional" vs "HS gauge-invariant couplings".

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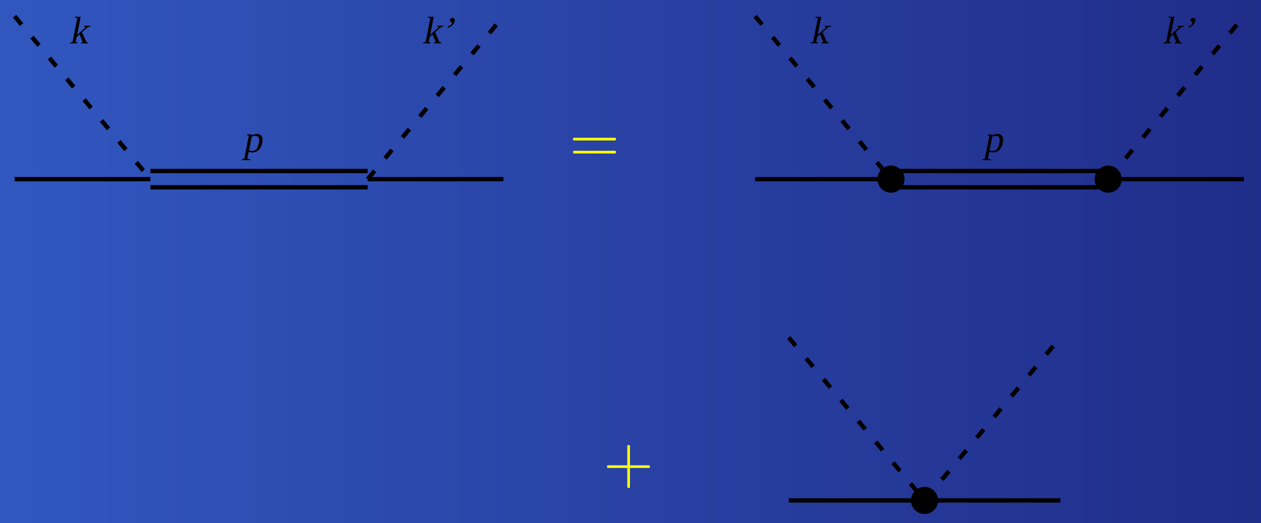
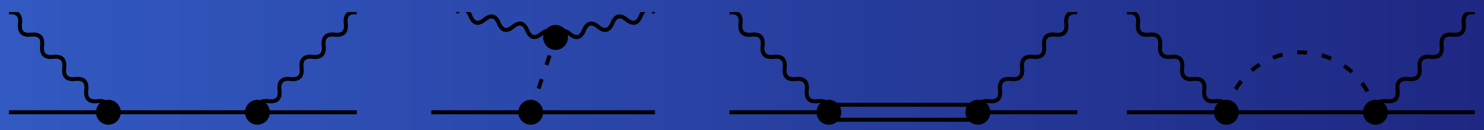
Not equal. For $m \neq 0$ can be chosen to be equal on shell.

Can be related by field redefinitions.



Ref: V. P., PLB 503 (2001).

Compton scattering



Compton LET for any s (forward scattering):

$$\begin{aligned} T_{fi} &= -\frac{e^2}{m}\,\vec{\varepsilon}\,' \cdot \vec{\varepsilon}\;\overline{u}'.u \\ &+ \frac{ie^2\,\omega}{4m^2}(g-2)^2\,(\vec{\varepsilon}\,' \times \vec{\varepsilon}) \cdot (\overline{u}'\,\vec{S}\circ u) \end{aligned}$$

$g = (\mu/s)(e/2m)^{-1}$ is the gyromagnetic ratio. LET is given by the Born terms. Evrth else begins at ω^2/M^3, M a mass scale.

Compton scattering

At low energy ($\omega \sim m_\pi$),

πN loops: $\omega^2 \frac{g_A^2}{(4\pi f_\pi)^2 \, m_\pi}$

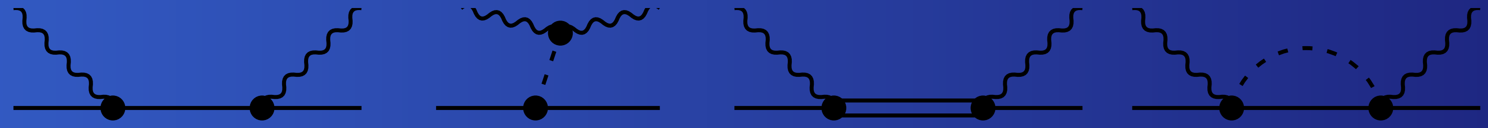
N^* exchange: $\omega^2 \frac{g_{\pi N N^*}^2}{m^2} \frac{1}{M_N^* - M_N}$

Short-range effects: ω^2 / Λ^3.

Two calculations of Compton scatterin

V. P. and D. Phillips, PRC (2003). An extension of ChPT:

Born $+$ one-πN loops $+\Delta$ with gauge-invariant (g_M, g_E).



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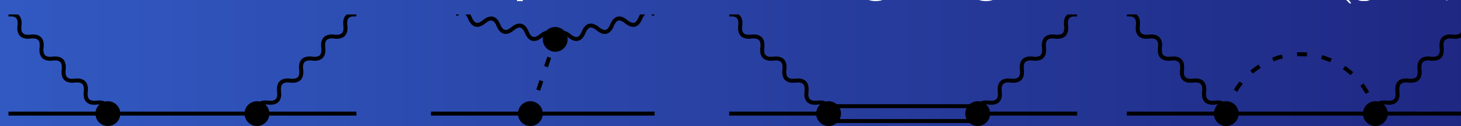

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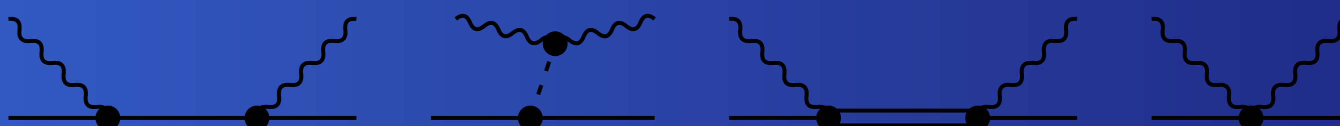

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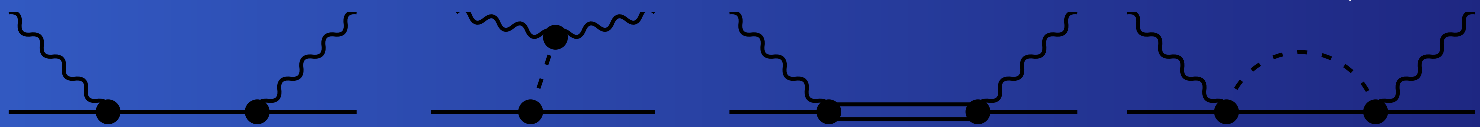

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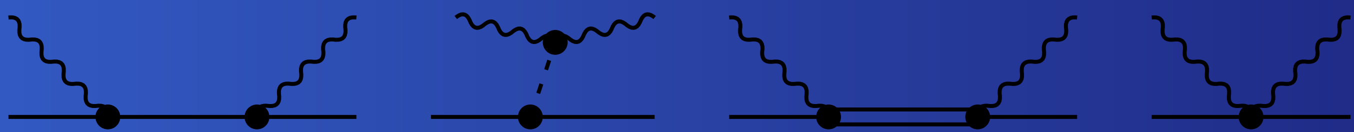
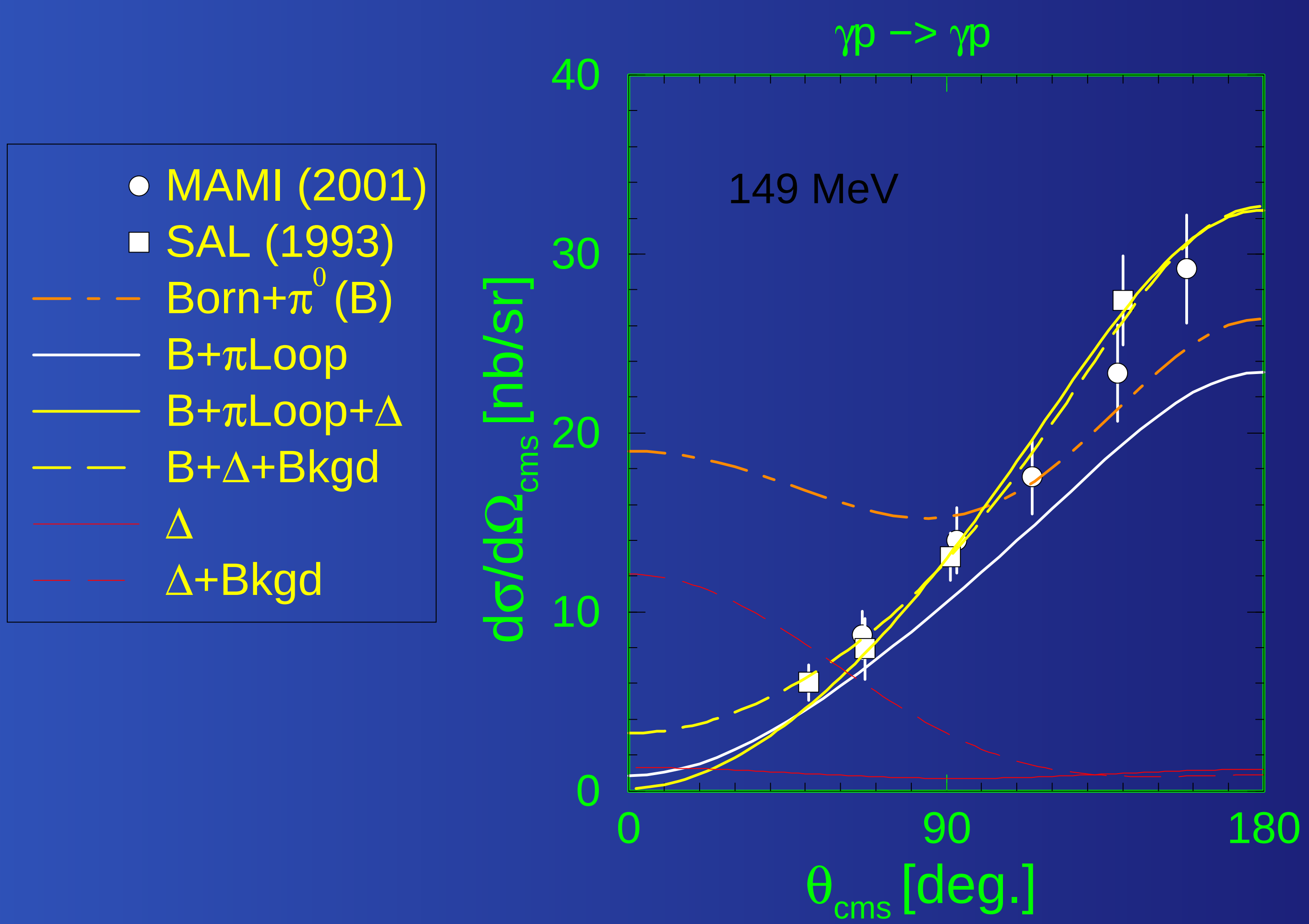


Difference: spin-1/2 bkgd contributions ($\gamma\gamma NN$ contact term)in ELA are replaced by the πN loops of ChPT. 2 less parameters.

Compton diff. cross-section



Dressing of the Δ

$$S^*_{\mu\nu} = S_{\mu\nu} + S_{\mu\alpha}\Sigma^{\alpha\beta}S^{\beta\nu} + \cdots = (S^{\mu\nu} - \Sigma^{\mu\nu})^{-1},$$

where, in general (for the inconsistent couplings),

$$\Sigma^{\mu\nu}(p) = (A(p^2)\not{p} + B(p^2))P^{3/2} + \sum_{ij}(C_{ij}(p^2)\not{p} + D_{ij}(p^2))P^{1/2}_{ij}$$

Thus, in general, the structure of S^* is very different from S.

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Thus, in general, the structure of S^* is very different from S.

Unless, $p \cdot \Sigma = 0$. Then,

$$S = \frac{1}{\not{p} - m}P^{(3/2)}, \quad S^* = \frac{1}{\not{p} - m - A(p^2)\not{p} - B(p^2)}P^{(3/2)}.$$

Conclusion

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- Strightforward Feynman rules, decoupling of lower-spin (unphysical) sector, easy derivation of relativistic amplitudes for any-spin hadron exchange (possible connection to Regge amplitudes), loops are OK.

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- Technically more complicated to deal with, fail in dressing, more parameters (to control bkgds), refit lower partial-waves every time a new HS state is included.
- Choice: consistent or inconsistent.