PHYS 894: "Gamma-Ray Astrophysics"

Fall 2008

Problem Set 2

1) Delta function approximation for Compton scattering

   a) Assume an ensemble of relativistic electrons with a distribution $N(\gamma)$ in electron energies, $E = \gamma\, m_e c^2$, which Compton-upscatters an isotropic radiation field of energy density $u_{\rm rad}$. Let's assume that this radiation field is monochromatic, i.e., all photons have the same photon energy ϵ_0. An electron with Lorentz factor γ will then, on average, scatter these photons to higher energies, $\epsilon_s \approx \gamma^2 \epsilon_0$. Neglect any statistical scatter in this relation, and balance the total energy loss of the electrons with the energy input into scattered radiation to develop a delta-function approximation for the Compton scattered spectrum.

   [40]

   b) Now, assume that we have standard non-relativistic first-order Fermi acceleration happening in a region of size $R = 10^{15}$ cm, and the time scale for escape of particles from this region is $t_{\rm esc} = 10^6$ s. Let's further assume that energy losses of particles in this region are dominated by Compton scattering in a radiation field of energy densith $u_{\rm rad} = 2 \times 10^{-2}$ erg cm^{-3}. Sketch the resulting equilibrium particle distribution. Give the spectral index (or indices) in different parts of the spectrum, and (if there is a break in the spectrum) the Lorentz factor of electrons at this break energy.

   [20]

   c) Based on your results from a) and b), sketch the resulting inverse-Compton X-ray/gamma-ray spectrum. Give the spectral index (or indices) of the emission and (if there is a break in the spectrum) the photon energy at which this break occurs.

   [20]

2) Estimate the total power output of a pulsar ($M = 1.4\, M_\odot$, $R = 10^6$ cm) with a period of $P = 0.3$ s and a period derivative of $\dot{P} = 10^{-15}$.

   [20]