Coupled diode resonators: a one-dimensional experimental model for glassy behavior

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We present results from an experimental one-dimensional set-up of coupled chaotic resonators that display glassy behavior. The system shows fluctuations from a spatiotemporal chaotic state to metastable traps. The distribution of trap times shows stretched exponential behavior, $\exp -(t/\tau_0)^\beta$, over 6 decades. Similar results can be obtained numerically using coupled logistic maps with a distribution of drive parameters. Even though the trap time distribution averaged over all sites shows a stretched exponential decay, we find strong heterogeneity in the site dynamics: individual sites display behavior ranging from power-law ($\beta = 0$) to pure exponential relaxation ($\beta = 1$).

The nature of the glass transition has received significant attention in the last few years following a number of theoretical, experimental and numerical developments. For example, Cugliandolo and co-workers have extended the application of the mode-coupling theory to systems without time-translation invariance [1]. Similarly, a number of experiments [2,3] have managed to probe the dynamics of glassy materials on length-scale relevant to study the problem of heterogeneities [4]. In spite of the considerable progress accomplished, the problem of the glass transition remains open; a clear description of the dynamics and thermodynamics of the phenomenon is still missing.

One approach to address these issues directly is to search for simple models that can capture the essence of the glass transition. Along these lines, many models of glassiness without disorder (but rather convoluted interactions) were introduced [5]. Another avenue is the search for dynamical models presenting a dynamics similar to that of glassy systems, i.e., stretched exponential relation. This can be done because although many of these coupled models are formally very different from structural glasses, it appears that the source of glassiness, either self-imposed or quenched in the rules, does not affect the nature of the phenomenon [1] (this might be the case for long-range interactions, however [5].) As it turns out, stretched-exponential relaxation is not so rare in dynamical systems. Frustrated high-dimensional cellular automata, for example, were found to display a stretched exponential relaxation [6]. Similarly, Ngai and Tsang showed that coupled oscillators, with infinite-range interactions, also behaved like supercooled liquids [7]. Although these models present a number of advantages over full-atom simulations, if only by the fact that they are much easier to program and faster to simulate, the high-dimensional space in which they are embedded does not allow us to develop a better physical image of the details of the dynamics. We are not aware of an experimental system showing glassy behavior in dynamical systems.

In this letter, we present evidence of stretched-exponential relaxation, $\exp\left[-(t/\tau_0)^\beta\right]$, in a well-controlled experimental setup of coupled diode resonators. This one-dimensional setup has been extensively studied and has been a test-bed for a number of questions in coupled dynamical systems [8,9]. With the building blocks of this experiment of macroscopic size, it it possible, for the first time in an experiment, to follow the dynamics of each site individually, providing a unique picture on the dynamics of a glassy system, including the existence and nature of heterogeneities. Finally, the experiment can be modeled fairly well by a one-dimensional array of coupled logistic maps [10], providing not only a strong case for the results but also a simple model to study glassy phenomena.

The experimental setup in shown in Fig. 1 of Reference [9]. It is composed of a linear chain of 256 diode resonators, all with a common ac drive and coupled to nearest neighbors with resistors. The peak forward currents through an uncoupled resonator follow the classic period-doubling route to chaos as the drive voltage is increased. The glassy behavior of the coupled system takes place well into the chaotic region of an uncoupled element.

The experimental setup can be compared with a much simpler coupled map lattice. Following Kaneko [11] and Johnson *et al.* [10], we take the logistic map, $f(x) = 4rx(1-x)$, and couple it to its nearest neighbors in a one-dimensional chain

$$f_i(t+\Delta t) = (1-\alpha) f_i(t) + \frac{\alpha}{2}\left[f_{i-1}(t) + f_{i+1}(t)\right]. \quad (1)$$

The parameter r of this model can be associated to the driving voltage of the experimental set-up. Because the properties of the diode resonators vary, we introduce heterogeneities in the system by making r a function of the site index, $r[i] \in [r-\delta, r+\delta]$.

The whole phase diagram of these coupled chaotic systems, as a function of driving, is complex and will be reported elsewhere [12]. We focus here on the phase transition found in both systems from the fully chaotic state, at high driving, to a less chaotic, metastable state as the drive is slowly decreased. In the condensed state, an individual site is in a noisy period-2 regime and can be coarse-grained as to its phase into a binary description.

For the coupling used, the ordered state for the coupled diode resonators is a two-up-two-down phase, i.e., the resonators are roughly in phase spatially with a period of 4; the corresponding state for the coupled maps has a period two for $\alpha = 0.1$.

From the chaotic state, however, the coupled oscillators do not fall into an ordered and stable state. They settle, instead, into a pattern of metastable traps, i.e., regions of space that keep their phase over some finite period of time. Figure 1 shows the time series for the 256-coupled diode resonators. Traps vary widely in size and period of stability.

The distribution of trap times, i.e., how long a site remains in a given phase, provides information on the stability of the system. In the chaotic regime, the distribution of trap times is exponential, indicating an absence of long-time correlation. In a coupled-map model without any disorder, this distribution starts following a power-law, as r is decreases, before freezing in a period-two mode. This power-law behavior, close to the transition, can be associated with that seen at a second-order phase transition in thermodynamical systems. As disorder is introduced, however, the distribution of trap times becomes more complicated.

In Fig. 2, we plot the distribution of experimental trap times averaged over a *single* site, something which is generally hard to measure in other experimental setups. The trap distribution is measured on a single site over 10^9 cycles, or 10 hours of experimental measurements. This distribution is very well approximated by a stretched exponential with an exponent $\beta = 0.40 \pm 0.05$ and $\tau_0 = 27 \pm 10$ drive cycles. The inset shows the trap-time distribution averaged over all the sites of a 2600-site chain of coupled maps for 10^8 steps, using the parameters given. Here again, the distribution is well approximated by a stretched exponential with $\beta = 0.50 \pm 0.05$ and $\tau_0 = 10 \pm 5$.

Figure 3 shows a time series for a 512-site simulation. For clarity, we have reversed the phase of every other site so that white represents one spatially alternating phase and black the other. Like for the experiment, we see a wide distribution of traps.

As is often the case for glasses, the exponent β, in the stretched exponential is not universal. It varies here almost continuously with decreasing driving from about 1.0 (exponential decay) to 0 (power-law). Figure 4 shows how β behaves for the experiment. At low external driving, β is zero and the system is either totally frozen or showing a power-law distribution of trap times. As the driving is increased, β jumps in a relatively narrow interval to a finite value of about 0.4 then climbs, with increasing drive, to 1.0, in the fully chaotic phase. We find the same behavior in the coupled map model.

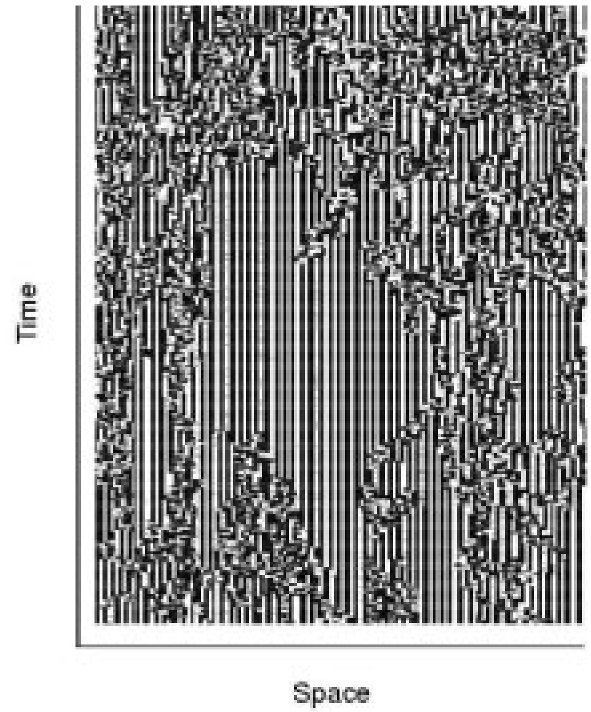

FIG. 1. Time series for the glassy regime of the coupled diode resonators as a binary representation of the 256 diodes over 4096 cycles. Since the system shows a fundamental period-two oscillation in time, we present here only the state at even time-steps.

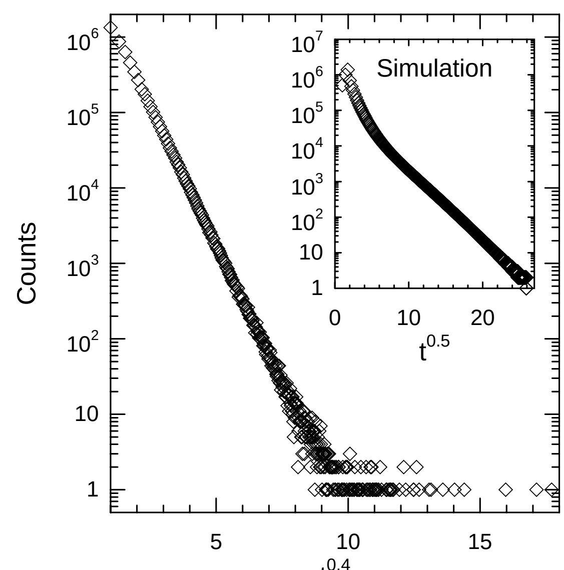

FIG. 2. Distribution of trap times measured over a single diode over a 10-hour period.. The curve shows the best fit to a stretched exponential with $\beta = 0.4$. Inset: Distribution of trap times for a 2600-site coupled map lattice with $\alpha = 0.1$, $r = 1.90$ and $\delta = 0.02$.

The relaxation time scale, defined by τ_0, cannot be categorized as easily. In Figure 4, we see that it follows

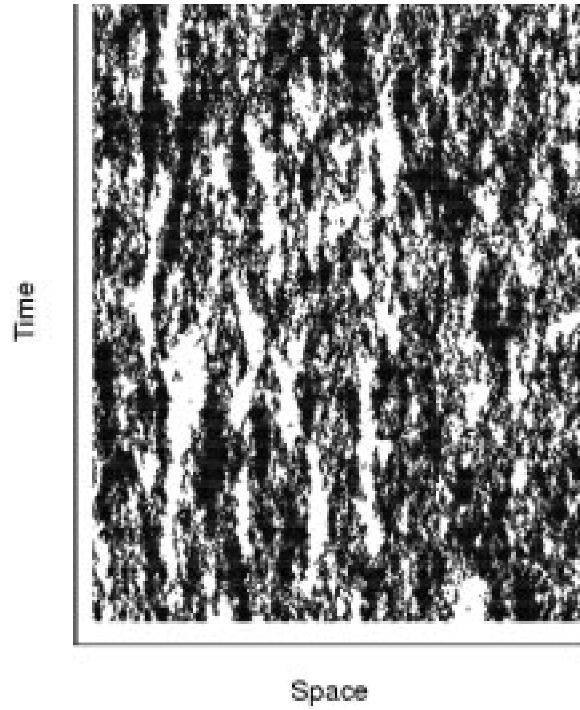

FIG. 3. Time series for a 512-site coupled maps with $\alpha = 0.1$ and $r = 1.90 \pm 0.02$. Since the coupled maps has as a period-two structure spatially, we have reversed the state of every other site for this figure. Since this system also shows a period-two phase in time, we present here only the state at even time-steps.

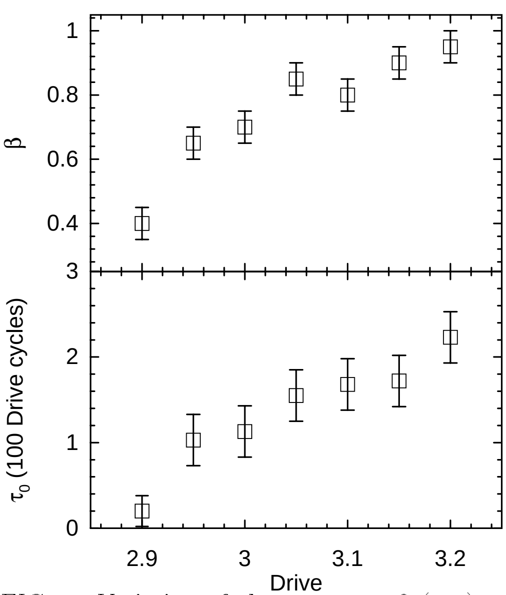

FIG. 4. Variation of the exponent β (top) and the time scale τ_0 (bottom) as a function of the drive voltage for the experimental setup. Similar behavior is obtained for the coupled map model.

β roughly. This leads to a time scale associated with the traps of about 100 drive cycles in the experiment and about 20 in the simulation with the parameters of Fig. 3. As a function of the size of the disorder, however, τ_0 can increase or decrease with driving. More remains to be done to understand this behavior. All these numbers are reproducible with different initial configurations and system sizes (for the simulation).

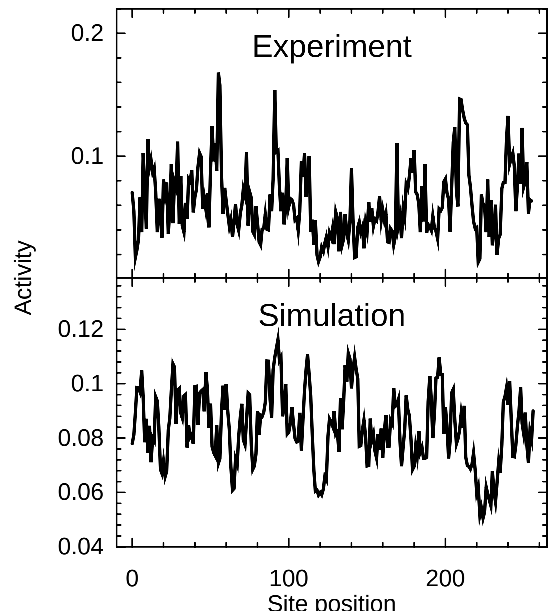

FIG. 5. Activity (fraction of the time steps where a site changes phase) as a function of site number for the 256 coupled diode-resonators (top) and a similar simulation (bottom).

Heterogeneities. The previous experimental correlations were obtained by time-averaging over single site. It is also possible to average over all sites on the experimental set up but with much less statistics. The results from the simulation show that the global trap time is also distributed according to a stretched exponential. We can now examine the question of heterogeneities. A number of constructions were proposed over the years to explain the origin of the stretched exponential in glassy materials [13]. Two main explanations arrive at opposite conclusions regarding the existence of these heterogeneities. (1) In the first case, the stretched exponential is achieved through a weighted sum of simple exponentials with a spread of relaxation times. Each of these exponentials would be attached to a region of space, hence, the heterogeneities. (2) The second theory asserts that on a crude enough length scale, the glass is homogeneous and all regions display a similar stretched-exponential behavior. Although the situation is not clear in the case of structural glasses, recent simulation of a spin-glass model demonstrated clearly that this system belongs to class (1) [14].

We can address this question directly in our systems. Figure 5 shows the activity associated with each site in the experimental and coupled map systems. Activity is defined as the number of times a site changes state normalized by the number of time steps. For both cases, the activity oscillates between 5 and 15 %, indicating clearly the presence of heterogeneities at the site level. The activity should be a measure of the local dynamics of the sites. Very active sites should have a dynamics close to that of the chaotic regime while the slow sites should show a trap-time distribution more in line with the frozen regime. Although mostly determined by the value of the local chaotic threshold, the variations in activity are also correlated with the local neighborhood as can be see in Fig. 5.

Figure 6 shows the relaxation for two sites that are at the extreme of activity. While the trap-time distribution for the cold site is consistent with a power-law, that of the hot site can be well described by a stretched-exponential with $\beta = 0.85$. For larger disorder or driving, the most active sites can also display pure exponential relaxation. Sites display therefore a continuous distribution of dynamics ranging from $\beta = 0$ to $\beta = 1.0$. Similar results are found for the experimental setup.

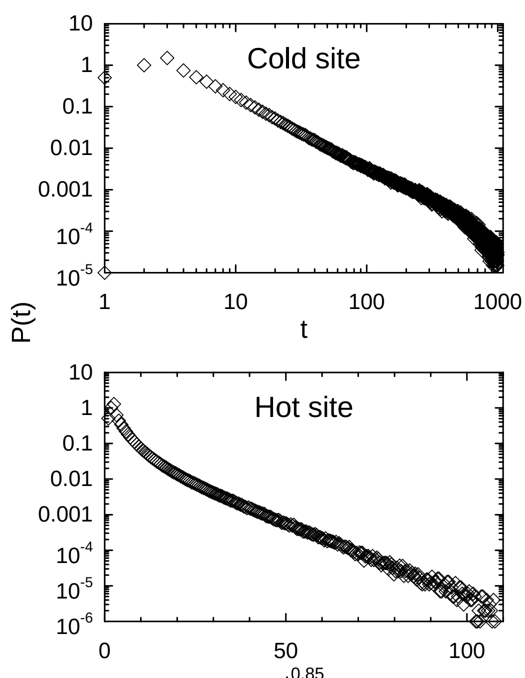

FIG. 6. Trap-time relaxation for a cold and a hot site in the coupled maps with the same parameters as in Fig. 3. The trap-time relaxation of the hot site follows a stretched exponential with $\beta = 0.85$ while that of the cold one is best described by a power-law.

Remarkably, for all external drivings, the spatial length scale associated with heterogeneities in both systems presented here is very short: in both experiment and simulation, spatial correlations decay exponentially, preventing any long-range spatial structure. For all purposes, the domains of heterogeneities are therefore of the order of length 1, covering a single site and its near neighbors. The overall behavior of our systems lead, however, to a more complex picture than what is normally presented: the stretched exponential can be obtained by summing a wide range of distributions and many types of relaxation can therefore take place in real glasses, with both simple exponential and more complex relaxation taking place in the relaxation domains.

In conclusion, we have demonstrated the existence, for the first time, of a one-dimensional dynamical experiment displaying glassiness, through a stretched-exponential relaxation. We also find that the heterogeneities giving rise to an overall stretched-exponential relaxation have a much wider range of dynamics than models generally consider. This set-up should allow us to continue exploring many ideas on the nature of the glass transition, providing one of the best models of glassy structure available today.

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