

Sample Technical Report:

Measurement and Error

Kellen Murphy
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Instructor: Kellen Murphy

Abstract

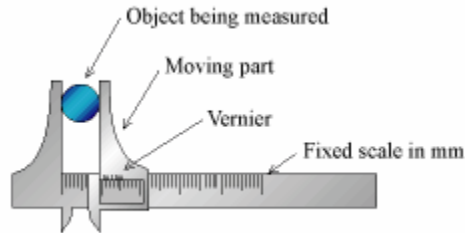
We compute the error associated with a manual measurement of the length and width of a metallic object of indeterminate size and shape. We analyze how errors arising in our measurements propagate and affect the accuracy of perimeter and area computations. We computed the area of our given metallic object to be $39.71 \pm 0.19 \text{ cm}^2$, and the value of the perimeter to be $25.79 \pm 0.06 \text{ cm}$.

Theory

Theoretical understanding of this lab is a twofold process. First, we must consider the experimental basis of our work: the reading of a Vernier scale attached to a caliper. A caliper is a handheld device for accurately measuring the physical dimensions of an object (i.e. the length of a piece of metal). The calipers that we are using achieve millimeter accuracy through the use of a Vernier scale as the means by which lengths are determined. In fact, the scale allows us to achieve an accuracy that this author feels is well within the acceptable accuracy for a by-hand measurement.

The Vernier scale was invented in 1631 by the French mathematician and physicist Pierre Vernier (1580 – 1637). [1] The Vernier scale revolutionized the science of measurement by allowing accuracy never before obtained with classical gradation schemes. One is able to achieve great accuracy in the reading of measurements off of the Vernier scale because of a simple principle derived by Portuguese mathematician Pedro Nunes. The scale is constructed such that a smaller subscale is spaced at a constant fraction of the main scale. For example, for a device used to measure length (such as the device we shall use in this experiment), a subscale would be marked such that each

demarcation was spaced at 9/10 of the spacing length on the main scale. We place the two scales together with aligned zeros, and hence have mismatched scalings whose variance is known.



[Figure I]

Moving the smaller scale (also called “the Vernier”), we are able to achieve an accurate measurement of the distance between the zero on the Vernier and the zero on the main scale, to an accuracy determined by the scaling marks. For example, on our trivial scale, we are able to achieve 0.01 cm accuracy.

When we determine the length and width of our metallic object (which is roughly rectangular), we want to obtain an estimate of the error. To do this we proceed through a given number of trials (in the case of this experiment, six trials will be used), and after performing our set of measurements, we will determine an arithmetic mean of the data. To do this, we merely sum the individual lengths (or widths) and divide by the total number of trials, i.e.

$$\bar{L} = \frac{\sum_{i=1}^N L_i}{N}. \quad (1)$$

Once an arithmetic mean is obtained from both the length and width data, we can proceed to compute the standard deviation for these data.

The *standard deviation* is a commonly used measure of how accurate given data is based on the spread in the data distribution. [2] Standard deviation, σ , of a given sample of data (also called the Root-Mean-Square, or RMS, deviation) is obtained by adding the squares of the *deviations* (i.e. $L_i - \bar{L}$), dividing by the total number of samples minus one, and taking the square root:

$$\sigma_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N - 1}}. \quad (2)$$

For this experiment, we will consider a separate quantity, the mean standard deviation:

$$\sigma_x^- = \frac{\sigma_x}{\sqrt{N}} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N(N - 1)}}, \quad (3)$$

since we're only interested in means for this experiment.

After obtaining values for the standard deviation of length and position, we are able to quote experimentally determined values for the length and width, so we now need only to understand how these errors propagate into calculations of area and perimeter. This requires two types of error propagations: additive (in the case of perimeter), and multiplicative (in the case of area).

Additively, error propagation is simple: for a given function z , which can be expressed as the sum of two values (with errors), we can write this expression generally as:

$$z = ax \pm by \quad (4)$$

Where x and y are the values with attributed errors and a and b are coefficients (hence allowing us to work generally). Given this scenario then, we can find the mean standard deviation in z from the equation:

$$\sigma_{\bar{z}} = \sqrt{a^2 \sigma_{\bar{x}}^2 + b^2 \sigma_{\bar{y}}^2}. \quad (5)$$

So, in the case of perimeter:

$$p = 2l + 2w, \quad (6)$$

so

$$\sigma_{\bar{p}} = 2\sqrt{\sigma_l^2 + \sigma_w^2}. \quad (7)$$

Multiplicatively, things are slightly more complicated, given a general multiplicative function:

$$z = ax^{+b} y^{\pm c} \quad (8)$$

the mean standard deviation of z is as follows:

$$\sigma_{\bar{z}} = \bar{z} \sqrt{b^2 \left(\frac{\sigma_{\bar{x}}}{\bar{x}} \right)^2 + c^2 \left(\frac{\sigma_{\bar{y}}}{\bar{y}} \right)^2}. \quad (9)$$

Hence, for the case of area:

$$A = l \cdot w, \quad (10)$$

so

$$\sigma_{\bar{A}} = \bar{A} \sqrt{\left(\frac{\sigma_{\bar{l}}}{\bar{l}} \right)^2 + \left(\frac{\sigma_{\bar{w}}}{\bar{w}} \right)^2}. \quad (11)$$

Experimental Details

The apparatus we shall use for this lab is the Vernier caliper, set for reading data with 0.01 cm accuracy. We'll use this caliper to measure the length and width of a somewhat rectangular shaped metallic plate. This plate is nearly rectangular; however there are slight aberrances which we seek to account for by reading the length and width

at six different points on the plate. This gives us a broad range of values that lets us get the most accurate portrait of the true area and perimeter of this plate. Only were we to extend the number of sample locations to infinity would we be given the true value of the area and perimeter; hence, we'll also give a quote of the error in our calculations based on the propagation of error in our measurements.

We expect errors to exist as a result of human error in measuring the lengths and widths of the plate. With any luck, these will errors will be systematic errors, and hence less impacting on the overall result. Unfortunately, random errors are more likely to occur as a result of our own experimental negligence. An example would of a gross, random human error would be measuring say, the width of the plate, and not holding the plate so that it is absolutely perpendicular to the caliper. This type of angular offset is likely to occur, and decreases the value of the length by a factor of the cosine of the angle between the plate and that plane which is perpendicular to the caliper head. This type of error is very easy to do, in fact, it occurs with every measurement as we likely do not have the manual dexterity to achieve and absolutely zero angle. Our only hope is that we are able to minimize the angle and hence minimize the impact on the length.

Another common source of error attributed to experimenter negligence would be simple misreading of the Vernier scale. Since the scale is not a digital device that can deliver guaranteed accuracy (we require our brains to do that), there arises error as a result of selecting the improper decimal from the scale (i.e. thinking that the scale aligns most with say, five, when it actually aligns most with six. Luckily, as we shall see in the data section, these sort of errors are typically within one standard deviation of the average value (most definitely within two standard deviations) and hence are generally accounted

for in the statistics. Also fortunately for us, these errors are just as likely to cancel each other out as they are to add together in the final result (a fact arising from the almost inherent systemic nature of this sort of error).

Furthermore, a major source of systematic error in this experiment is the aforementioned problem of using a discrete set of measurement points. Since we're using quite a limited set of data points (six) for each measurement, we can expect the confidence level of our final results to not be that high. We were to use more data points (perhaps in "future work") we could expect our confidence in the final results for perimeter and area to rise.

Results and Discussion

Table I contains the raw data from the six measurements of length and width, the computed values for the mean deviation of each of the individual runs, and the mean deviations squared. These deviations will be used in conjunction with Equation 3 to compute the mean standard deviation of the length.

| L_i | $L_i - L_{avg}$ | $(L_i - L_{avg})^2$ | W_i | $W_i - W_{avg}$ | $(W_i - W_{avg})^2$ |
|-------|-----------------|---------------------|-------|-----------------|---------------------|
| 7.810 | 0.005 | 0.0000 | 5.090 | 0.002 | 0.0000 |
| 7.820 | 0.015 | 0.0002 | 5.100 | 0.012 | 0.0001 |
| 7.760 | -0.045 | 0.0020 | 5.100 | 0.012 | 0.0001 |
| 7.800 | -0.005 | 0.0000 | 5.110 | 0.022 | 0.0005 |
| 7.830 | 0.025 | 0.0006 | 5.070 | -0.018 | 0.0003 |
| 7.810 | 0.005 | 0.0000 | 5.060 | -0.028 | 0.0008 |

[Table I – Results for six measurements of length and width of metal plate, as well as the computed values for the mean deviation of each of the individual runs, and the mean deviations squared.]

We are now able to compute the standard deviation of the mean of the length and the width, according to Equation 3. When we perform these calculations, we obtain:

$$\begin{aligned}\sigma_{\bar{l}} &= 0.0243 \text{ cm} \\ \sigma_{\bar{w}} &= 0.0194 \text{ cm}.\end{aligned}$$

Lastly, utilizing Equation 9 and Equation 11, we can determine the standard deviation in the computed values of perimeter and area:

$$\begin{aligned}\sigma_{\bar{p}} &= 0.0622 \text{ cm} \\ \sigma_{\bar{A}} &= 0.1955 \text{ cm}^2.\end{aligned}$$

Hence, we quote these values as the average plus/minus the standard deviation and obtain our final experimental results:

$$\begin{aligned}l &= 7.805 \pm 0.024 \text{ cm} \\ w &= 5.088 \pm 0.019 \text{ cm} \\ p &= 25.788 \pm 0.062 \text{ cm} \\ A &= 38.714 \pm 0.196 \text{ cm}^2.\end{aligned}$$

Conclusions

In this lab, we discovered the ways in which errors propagate from measurements through to final results. The errors which we found for our calculation of area and perimeter were quite small (.51% and 0.24%, respectively); quite smaller than we could have imagined our results would be from only six data points for each set of data. We attribute this to the overall human error in measuring the device, be that error from angular deviation of merely misreading the Vernier scale. Nevertheless, we choose to trust our equations and expect that our estimates are somewhat low because of the experimenter.

We find that the results obtained do, however, verify the propagation of error into the computations of area and perimeter quite nicely. We obtained error estimates for the length and width and these translated quite nicely into errors in the perimeter and area. We also find, quite expectedly, that the error in the computation of the area is much larger than the error in the computation of perimeter (47% higher) as expected, since errors combined multiplicatively should give greater overall error than when combined additively.

Lastly, the results of this experiment do tend to highlight the many sources of error one can encounter in a laboratory environment. We have, for sake of brevity, only discussed a few of the most important sources of error, but it is apparent that the number of sources is nearly limitless, and therefore it is impossible to come up with a completely perfect concept of how much error is in one's experiment. Hopefully, we have accounted for the most egregious errors, allowing for the fact that any remaining errors are likely less important. Nevertheless, one can argue that were we performing a mission critical experiment, the results of our experiment are still immensely helpful in that they tell us we need to put more time and effort into... wait for it... producing a better experimental setup. As is often the case in physics, we see here perfectly how one experiment opens the door for another (presumably more expensive) experiment.

Bibliography

[1] Wikipedia, “Vernier Scale.” http://en.wikipedia.org/wiki/Vernier_scale

[2] Lichten, William. Data and Error Analysis, 2nd Edition. New Jersey: Prentice Hall, 1988.