Schrödinger Equation

1. (4 p)
In the region $0 \leq x \leq a$, a particle is described by the wave function $\psi_1(x) = -b(x^2 - a^2)$. In the region $a \leq x \leq w$, its wave function is $\psi_2(x) = (x - d)^2 - c$. For $x \geq w$, $\psi_3(x) = 0$.

(a) By applying the continuity conditions at $x = a$ find $c$ and $d$ in terms of $a$ and $b$.
(b) Can you find $w$ in terms of $a$ and $b$?

2. (3 p)
In a certain region of space, a particle is described by the wave function $\psi(x) = Cxe^{-bx}$, where $C$ and $b$ are real constants. By substituting this into the Schrödinger equation, find the potential energy in this regime and also find the energy of the particle.
(Hint: Your solution must give an energy that is a constant everywhere in this region, independent of $x$.)

3. (2 p)
The Schrödinger equation for a particle with potential energy $V(x)$ may be written as

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar} [V(x) - E] \psi = 0 \quad (1)$$

Using this equation, show that if the energy is less than the minimum value of $V(x)$ the wave function and its second derivative always have the same sign and the function $\psi$ can not be normalized.

4. (3 p)
Consider a finite well for the case that the energy $E$ is greater than the well depth $V_0$. Show that in this case the Schrödinger equation for the region outside the well can be written as

$$\frac{d^2\psi(x)}{dx^2} + k^2 \psi(x), \quad (2)$$

with

$$k = \left[\frac{2m(E - V_0)}{\hbar^2}\right]^{1/2}, \quad (3)$$

where $k$ is a real number. What is the general form of the solution of Eq. (2).