Assignment III: Bessel-functions and Roots

Due 9/15/2014

1. Bessel-functions via Recursion

(a) Write a program to calculate \( J_l(x) \) that will give ‘good’ values for the first 25 \( l \) values for \( x = 0.1, 1.0, 10. \) (‘Good’ here means a relative error of \( \approx 10^{-6}(10^{-14}) \) for single (double) precision). [Pick one!]

(b) Try with both, upward and downward recursion, but do not try too hard for upward recursion. [Hint: Using single precision shows the error effects more quickly.]

(c) Give results of the downward recursion for different, large values of the starting \( l \), showing the convergence and stability of your results.

(d) Compare the upward and downward recursion methods, printing out \( l, J_l^{(up)} \), and \( J_l^{(down)} \), and the relative difference

\[
\frac{|J_l^{(up)} - J_l^{(down)}|}{|J_l^{(up)}| + |J_l^{(down)}|}
\]

(e) The errors in the upward recursion depend on \( x \), and for certain values of \( x \), both up and down recursions give similar answers. Explain the reason for this and what it tells about your program.

2. Finding Roots

Find the positive roots of

\[
x^2 - 4x \sin x + (2 \sin x)^2 = 0,
\]

using

1. the bisection method
2. the Hybrid/Newton-Raphson methods based on the Secant
3. the Brent method

Discuss the rate of convergence of the methods and their robustness. When running your code, consider if you need prior knowledge of the approximate position of the zero.

3. The neutron-proton system (deuteron) moving in a box potential

See Hjorth-Jensen Chapter 4, problem 4.3.