Assignment VII: Project I

Due 10/13/2014

Please put your final tar-file at the due date by 8:15 am in the same directory as your homework. Please name your tar-file Project1.tar.

1. Woods-Saxon Function
The function
\[ f(r) = \frac{1}{1 + e^{\frac{r-c}{a}}} \]  
(1)
is known by the name of “Woods-Saxon” function in nuclear physics. The constant \( c \) is related to the nuclear radius, and \( a \) is the surface thickness of the varying density. The “volume” of \( f(r) \) can be written as series,
\[ V \equiv 4\pi \int_0^\infty \frac{r^2 \, dr}{1 + e^{\frac{r-c}{a}}} = 4\pi a^3 \left[ \frac{1}{3} \left( \frac{c}{a} \right)^3 + \frac{\pi^2 c}{3} a + 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^3} e^{-\frac{nc}{a}} \right]. \]  
(2)

1.1 Evaluate the integral by Simpson, Gauss-Legendre and Gauss-Laguerre methods and achieve an accuracy of 6 significant figures for each method. Give in your write up the numerical answer together with the number of integration points needed, and convince yourself that you have for each method a convergence to the required amount of significant figures.

1.2 Then consider the right-hand side of the equation and calculate the sum. How many terms in the sum do you need to achieve 6 significant figures?

1.3 Ideally, the 4 different methods you used to calculate \( V \) should give the same result. Discuss your calculations.

1.4 Calculate \( \langle r^2 \rangle^{\frac{1}{2}} \) and \( \langle r^4 \rangle^{\frac{1}{2}} \) as well. Here
\[ \langle r^n \rangle = \frac{4\pi}{V} \int_0^\infty \frac{r^{n+2} \, dr}{1 + e^{\frac{r-c}{a}}} \]  
(3)
Use the constants \( c = 2.0 \) and \( a = 0.5 \).

2. Scattering Cross Sections
Cross section data were taken at 7 angles, listed in Table 1. Plot the data.

2.1 Calculate the integrated cross section
\[ \sigma_I \equiv \int d\Omega \sigma(\cos \theta) = 2\pi \int_{-1}^{1} dx \sigma(x) \]  
(4)
Here you need to be very careful since you need to extrapolate to \( \theta \rightarrow 0 \) and \( \theta \rightarrow 180 \) deg. Come up with a strategy to do so and explain it in your writeup. You should assume
Table 1: Cross Section Data

<table>
<thead>
<tr>
<th>$\theta$ [deg]</th>
<th>$\sigma(\theta)$ [mb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>8.447</td>
</tr>
<tr>
<td>35</td>
<td>7.948</td>
</tr>
<tr>
<td>55</td>
<td>6.624</td>
</tr>
<tr>
<td>75</td>
<td>5.102</td>
</tr>
<tr>
<td>90</td>
<td>4.000</td>
</tr>
<tr>
<td>140</td>
<td>1.523</td>
</tr>
<tr>
<td>155</td>
<td>1.196</td>
</tr>
</tbody>
</table>

that the cross section is a continuous function. Which integration method will be most helpful?

2.2 Experimental analysis of cross sections often consists of trying to represent the cross section in terms of Legendre polynomials as

$$\sigma(\cos \theta) = \sum_n a_n P_n(\cos \theta)$$

(5)

with

$$a_n = \frac{2n + 1}{2} \int_{-1}^{1} dx \sigma(x) P_n(x)$$

(6)

This is usually called partial wave analysis, and $a_n$ is the coefficient for the nth partial wave.

2.3 To progress further, you need to write a code calculating the Legendre polynomials $P_n(x)$. They can be obtained efficiently by the recursion relation

$$(n + 1)P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x)$$

(7)

with $P_0(x) = 1$ and $P_1(x) = x$. The integral of the square of $P_n(x)$ is given by

$$\int_{-1}^{1} dx P_n^2(x) = \frac{2}{2n + 1}$$

(8)

You may want to use Eq. (8) to check the accuracy of your recursion relation.

2.4 Determine coefficients $a_n$. Which coefficients are meaningful within the accuracy of your calculation? Give a table of the coefficients you plan to use for describing the cross section, and argue how high you want to go in $n$ to represent the sum in Eq. (5).

2.5 Use the coefficients $a_n$ to write a subroutine to calculate $\sigma(\cos \theta)$ at 5 selected angles not in your data set. Create a plot of the data together with an interpolation on the data. Then plot the cross section values you obtained using in Eq. (5) into the plot. Ideally your points should be on or close to your interpolated line.