Phys. 612: Homework I

due April 6, 2005

1. (a) (2 p) Suppose the neutron-proton force in the deuteron (ground-state) can be described by the potential

\[ V = V_1(r) + V_2(r)s_1 \cdot s_2 \]

where \( s_1 \) and \( s_2 \) are the spin operators of the proton and neutron, and the potentials \( V_1 \) and \( V_2 \) depend only on the neutron-proton separation \( r \). What are the good quantum numbers of the deuteron?

(b) (4 p) If the additional term (called tensor force)

\[ V' = V_3(r) \left( \frac{3(s_1 \cdot r)(s_2 \cdot r)}{r^2} - s_1 \cdot s_2 \right) =: V_3(r)S_{12} \]

is added to the above potential \( V \), what are now the good quantum numbers?

2. (4 pts)
Show that any positive integral power of each of these operators

\[ (s_1 \cdot s_2) \quad \text{and} \quad S_{12} = \frac{3(s_1 \cdot r)(s_2 \cdot r)}{r^2} - s_1 \cdot s_2 \]

and also any product of these powers can be represented in the form of a linear combination of these operators and a unit matrix.

3. Consider two particles of masses \( m_1 \neq m_2 \) interacting via the Hamiltonian

\[ H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{1}{2}m_1\omega^2 x_1^2 + \frac{1}{2}m_2\omega^2 x_2^2 + \frac{1}{2}K(x_1 - x_2)^2 \]  \hspace{1cm} (1)

(a) (6 pt) Find the exact solutions.

(b) (3 pt) Sketch the spectrum in the weak coupling limit \( K \ll \mu\omega^2 \), where \( \mu \) is the reduced mass.