Phys. 611: Homework VIII

due March 2, 2005

1. Electromagnetic Interaction

The classical Hamiltonian for a particle of charge $e$ moving in an electromagnetic field with vector potential $A$ and a scalar potential $\Phi$ is given by

$$H = \frac{1}{2m} \left( \vec{p} - \frac{e}{c} \vec{A} \right)^2 + e\Phi$$

(a) Derive the corresponding quantum mechanical Hamiltonian and the corresponding Schrödinger equation (in position space) for the motion of a particle of charge $e$. (4 pts)

In order to do so, consider the operator

$$\Omega = \left( \vec{p} - \frac{e}{c} \vec{A} \right)^2 \equiv \vec{p}^2 - \frac{e}{c} (\vec{p} \cdot \vec{A} + \vec{A} \cdot \vec{p}) + \frac{e^2}{c^2} \vec{A}^2$$

and show that $\Omega$ is hermitian.

(b) Classical electrodynamics is invariant under gauge transformations

$$\begin{align*}
A & \rightarrow A' = A + \nabla \chi \\
\Phi & \rightarrow \Phi' = \Phi - \frac{1}{c} \frac{\partial \chi}{\partial t}
\end{align*}$$

where $\chi(\vec{r}, t)$ is an arbitrary function. This means that the observables

$$\begin{align*}
\vec{B} & = \nabla \times \vec{A} \\
\vec{E} & = -\nabla \Phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}
\end{align*}$$

are invariant under the transformation group (3). Show that the corresponding Schrödinger equation is gauge invariant, if in addition the wave function is transformed as

$$\Psi \rightarrow \Phi' = e^{i\alpha} \Psi$$

with $\alpha = \frac{e}{\hbar c} \chi$. (6 pts)