1. (8 pts)
The normalized, coordinate-space plane wave solution of the Dirac equation is given as
\[ \Phi_s^{(\pm)}(x) = e^{\pm ip \cdot x} u_s^{(\pm)}(p), \] (1)
where the space-time dependence is in the exponential, the 4D spin dependence in the spinor, and \( p \) is not an operator.

When this plane wave solution is substituted into the coordinate-space Dirac equation one obtains the momentum-space Dirac equation for the free particle spinors \( u_s^{(\pm)}(p) \):
\[ (\gamma^\mu p_{\mu} \mp m)u_s^{(\pm)}(p) = 0 \] (2)

(a) Prove the relation (2) starting from (1) for \( u_s^{(+)}(p) \) and \( u_s^{(-)}(p) \).

(b) The adjoint spinor \( \bar{u}_s^{(\pm)}(p) = u_s^{(\pm)\dagger}(p) \gamma^0 \) satisfies the transpose Dirac equation
\[ \bar{u}_s^{(\pm)}(p)(\gamma^\mu p_{\mu} \mp m) = 0 \] (3)
Prove the above equation (3) for both cases.

2. (10 pts)
When manipulating solutions of the Dirac equation, it is often useful to combine groups of them into a simple operator. To do this, define the projection operators
\[ \Lambda_{\pm}(p) = \frac{\pm \gamma^\mu p_{\mu} + m}{2m} \] (4)

(a) Show that these operators are projection operators, i.e. fulfill
\[ \Lambda_+^2(p) = \Lambda_+(p) \]
\[ \Lambda_-\Lambda_+ = \Lambda_+\Lambda_- = 0 \]
\[ \Lambda_+ + \Lambda_- = 1 \] (5)

(b) Show that these operators have the desired effect when applied on the Dirac spinors:
\[ \Lambda_{\pm}(p)u_s^{(\pm)}(p) = u_s^{(\pm)}(p) \]
\[ \Lambda_{\mp}(p)u_s^{(\pm)}(p) = 0 \] (6)
3. (8 pts)
In relating field theoretic amplitudes to nonrelativistic potentials, it will be necessary to know nonrelativistic limits of the bilinear covariants. Determine the leading order terms in $p/m$ for

\begin{align}
(a) & \quad \bar{a}_s(p') \gamma^0 u_s(p) \\
(b) & \quad \bar{a}_s(p + q) \gamma^5 u_s(p) \quad (7)
\end{align}