Spin-dependent transport of electrons in the presence of a smooth lateral potential and spin-orbit interaction

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We describe theoretically the process of multibeam reflection in a two-dimensional electron system with a lateral potential barrier. Due to spin-orbital interaction, the reflection process leads to the formation of three beams with different spin polarizations. The efficiency of spin conversion can become small for smooth lateral barriers. Nevertheless, we demonstrate that the spin-conversion effect remains strong for realistic lithographical potentials and spin-orbit interactions in etched lateral nanostructures. The system with a lateral barrier suggests useful applications as a spin-filtering device. The expected quasiclassical adiabatic behavior without spin conversion is found in the system with a very strong spin-orbit interaction. We also consider the quasiclassical motion of electrons in a system with boundaries in a magnetic field and two magnetic focusing geometries.

INTRODUCTION

Mobile electrons in mesoscopic and nanoscopic structures experience spin-orbit interaction (SOI) and therefore the translational and spin motions of an electron become coupled. The most common type of measurements in semiconductor nanostructures concerns electric currents. However, since the SOI in the most common semiconductors is weak, electric-current measurements can be relatively insensitive to spins. There are several methods to enhance the spin effects in the electric-current measurements, such as the use of ferromagnetic leads to the semiconductor quantum well,1 driving currents through multibarrier structures, waveguides, channels, or quantum dots,2 etc.

In the absence of electric and magnetic fields, electrons with opposite spins move along a straight line. However, when electric and magnetic fields are present, electron trajectories depend on a spin state and the SOI effects in the electric conductance become enhanced. Recently, several mechanisms of spatial separation of electron beams with different spin orientations in ballistic lateral nanostructures have been proposed. The spin-polarized beams can be obtained by using spin-dependent reflection from a lateral barrier,3 a spatially varied SOI interaction,4 and cyclotron motion.5 In this paper, we focus on physical properties of spin-dependent reflection in a two-dimensional (2D) electron system with a lateral barrier, following the method proposed in Ref. 3.

Here we describe the electron reflection process in a 2D gas with a SOI. This process has a multibeam character and can be utilized for spin filtering in mesoscopic 2D systems with ballistic electron beams [Figs. 1(a) and 1(b)]. In such structures, an electron beam injected from the incoming window becomes reflected from a lateral barrier, and then leaves the system through the outgoing aperture. A weak magnetic field serves as a tool to focus the electron beam to the outgoing aperture. Due to the SOI, the incoming beam becomes split into three beams at the lateral barrier. This effect comes from the simple kinematics and will be explained below. In this paper, we show that the effect of multibeam reflection is strong if the lateral barrier is sharp enough. In the case of a very smooth potential, the spin-dependent reflection vanishes. Here we calculate the reflection coefficient for etched semiconductor nanostructures and show that the multibeam reflection effect is strong for realistic lateral barriers with relatively smooth potential profiles.

I. MODEL

The motion of single electrons in a 2D system in the presence of electric and magnetic fields is described by the Hamiltonian

$$\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m_e} + U(\vec{r}) + \hat{V}_{SO},$$

where $\vec{r}=(x,y)$ is the in-plane radius vector, $U(\vec{r})=e\varphi(\vec{r})$, $\hat{\vec{p}} = \hat{\vec{p}}-(e/c)\hat{A}$, and $\hat{\vec{p}}$ is the in-plane momentum operator; $\hat{\vec{A}}$ and $\varphi$ are the vector and scalar potentials, respectively; $\hat{V}_{SO}$ represents the SOI. For the SOI, we assume the Bychkov-Rashba inversion asymmetry mechanism induced by in-plane and perpendicular electric fields.6

We now consider the reflection process for a one-dimensional barrier $U(x)=e\varphi(x)$ in an asymmetric quantum well. In this case, the SOI operator takes a form

$$\hat{V}_{SO}(x) = \frac{\alpha_{SO}}{\hbar}(\hat{\sigma}_x \hat{P}_y - \hat{\sigma}_y \hat{P}_x) + \frac{\gamma dU(x)}{\hbar} \hat{\sigma}_z \hat{P}_x,$$

where $\hat{\sigma}_i$ are the Pauli matrices. The first term in Eq. (2) originates from the perpendicular electric fields in an asymmetric quantum well, and the second from the in-plane electric field. The material parameter $\gamma$ describes the strength of the SOI. If the SOI arises from the lateral and perpendicular built-in electric fields, the parameter $\alpha_{SO}=-e\gamma F_z$, where $F_z$ is the strength of the perpendicular built-in electric field. The
the spin is perpendicular to the momentum due to the SOI. The SOI dimensionless parameter 

\[ \delta = \frac{\Delta_{SO}}{E_F}, \]

where \( \Delta_{SO} \) is the spin splitting at the Fermi energy. Since the spin state is not conserved in the reflection process, the reflected wave has two components propagating at different angles: 

\[ \Psi^{\text{ref}} = A(+)\Psi_q^+ + A(-)\Psi_q^-, \]

where the momenta in the reflected waves, \( \tilde{q}_f = (q_x, k_x) \) and \( \tilde{q}_s = (q_x, k_x) \), are determined by the kinematics conditions sketched in Fig. 2(c) (for details, see Ref. 3). In Fig. 2(c), we also show the scattering angles \( \theta_{\text{inc}} \) and \( \theta_{\text{scat}} \) for the processes \( \rightarrow \rightarrow \) and \( \rightarrow \rightarrow \). The processes \( \rightarrow \rightarrow \) and \( \rightarrow \rightarrow \) conserve the angles: \( \theta_{\text{inc}} \rightarrow \theta_{\text{scat}} \rightarrow \theta_{\text{inc}} \). Note that the incident wave \( \Psi^i \) has a critical angle \( \theta_c \), at which the second scattered beam vanishes; in the limit \( \delta \ll 1 \), we obtain \( \theta_c = \pi/2 - \sqrt{\delta} \). For the angles \( \theta > \theta_c \), the electron wave function contains a wave localized near the barrier and propagating in the \( -y \) direction. Above, we used the typical parameters of InSb quantum wells, \( m_e = 0.014m_0 \), \( \Delta_{SO} = 10^6 \text{ meV cm} \), and \( \gamma = 1 \times 10^{-14} \text{ cm}^2 \).}

### III. QUASICLASSICAL MOTION

In the presence of weak and smooth fields, the motion of an electron is quasiclassical and is given by the usual equations,

\[ \dot{\vec{r}} = \vec{F}, \quad \dot{\vec{p}} = \frac{1}{\hbar} \frac{\partial \Psi}{\partial \vec{r}}. \]

where \( \vec{F} \) is the classical force and \( \alpha = \pm \) is the spin-state index. This approximation implies that the electron does not make transitions between the spin states \( \alpha = \pm \).

It is easy to solve the above quasiclassical equations in the uniform magnetic field \( B \). This solution will allow us to analyze the electron motion outside the barrier. In the presence of SOI, the cyclotron radii for the states \( \pm \) at the Fermi level become slightly different, \( R_{c,\pm} = \hbar k_{F,\pm} m_e / \omega_a \), where \( k_{F,\pm} \) are the Fermi wave vectors and \( \omega_a = |e|B / m_e c \). If the magnetic field is weak enough, it does not affect the reflection process.
metrical resonances. As the magnetic structure is relatively weak, experimentally, this system cannot be described by the quasiclassical equations in the vicinity of the classical turning point. In this case, the incident wave $\Psi_{\text{inc}}$ is always converted into the state $\Psi_{\text{out}}$. This follows from the fact that the spin and translational motions in the Hamiltonian (1) are separated for $\theta = 0$. The quasiclassical approach is applicable to a given point of trajectory if the SOI splitting at this point $\Delta_{\text{SO}} = 2\alpha_{\text{SO}} k$ is larger than $eF_{\perp} \lambda = eF_{\perp} \omega_c k$. The minimum spin splitting along a trajectory corresponds to the classical turning point $(k_x = 0)$ and is given by $\Delta_{\text{SO}, \text{min}} = 2\alpha_{\text{SO}} k_y$. An entire trajectory can be described with the quasiclassical approach for sufficiently large incident angles when $\Delta_{\text{SO}, \text{min}} = 2\alpha_{\text{SO}} k_y > eF_{\perp} \omega_c / k_y$. If the SOI splitting $\Delta_{\text{SO}}$ at some point of trajectory is comparable with the electric-field energy $eF_{\perp} \omega_c / k_y$, the incident beam $\Psi_{\text{inc}}$ is partially converted into $\Psi_{\text{out}}$ and vice versa. In the regions with a strong SOI splitting ($\Delta_{\text{SO}} = 2\alpha_{\text{SO}} k_y > eF_{\perp} \lambda$), the wave function can be written in a quasiclassical fashion. To write down analytical equations for the wave functions, we now neglect the last term in the spin operator (2), assuming a smooth lateral potential. For the incoming + wave, we obtain

$$\Psi = \psi_{\text{inc}} + \psi_{\text{out}},$$

$$\psi_{\text{inc}} = \frac{A_+}{\sqrt{2\nu_{A,+}(x)}} \left( e^{ik_x y + f k_x y} dx \right),$$

$$\psi_{\text{out}} = \frac{A_+}{\sqrt{2\nu_{A,+}(x)}} \left( e^{ik_x y + f k_x y} dx \right) + \frac{A_-}{\sqrt{2\nu_{A,-}(x)}} \left( -e^{ik_x y + f k_x y} dx \right),$$

where the wave vectors $k_x \pm = \sqrt{x^2 + a^2}$ are given by the equation $E_{k_x} = \pm \omega_c$. The y component of the momentum is conserved and $k_y < 1$; $\nu_{A,+}(x) = (1/\hbar) \partial E_{k_x} / \partial k_y \approx \nu_{A,+}(x)$, $|A_+|^2 = 1$, $\tan \phi_{\text{inc}}(x) = |k_x|/k_y$, and $\tan \phi_{\text{out}}(x) = -|k_x|/k_y$. The amplitudes in Eq. (5) obey the conservation law $\cos(\theta)|A_+|^2 = \cos(\theta)|A_+|^2 = \cos(\theta, A_+)|A_-|^2$; the lower index in $A_+(-)$ denotes a type of incoming wave. For the $x$ component of the momentum, we have

$$k_x = \sqrt{m^2 (\alpha^2 + 2mF_{\perp} - U(x)) + a^2} - k_y^2.$$

We now consider the case of entirely quasiclassical trajectory in a sense of the inequality $\Delta_{\text{SO}, \text{min}} = 2\alpha_{\text{SO}} k_y > eF_{\perp} \omega_c / k_y$; this inequality implies $k_y < 0$. Then, $A_+(-) \approx 0$.

**FIG. 3.** Different types of electron trajectories in the presence of a smooth potential barrier (a,b,c). (d) Sketch of the barrier.
in Eq. (5) and the outgoing wave remains in the state + [Fig. 3(c)]. Equations (5) diverge at the classical turning point where \( k_y(x) = 0 \) and \( v_y(x) = 0 \). Near the classical turning point for the + wave, \( x_{0+} \), the Hamiltonian can be approximated as

\[
\hat{H} \approx -\frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial x^2} + \frac{\hbar^2 k_y^2}{2m_e} + U(x_{0+}) - eF_0(x - x_{0+}) + \alpha_{SO} \sigma \hat{k}_y,
\]

(6)

where \( F_0 = F_0(x_{0+}) = -e^{-1} dU/dx \). The spin and translational variables in Eq. (6) can be separated. The solution has the well known form \( \Psi = \text{const} \times \text{Ai}[x - x_{0+}/l_0] \), where \( \text{Ai} \) is the Airy function and \( l_0 = (\hbar^2/2m_e F_0)^{1/3} \). Then, by using the asymptotic behavior of the Airy function, we obtain

\[
\Psi^\text{in}_+ = \frac{1}{\sqrt{2 \nu_{y+}(x)}} \left( \begin{array}{c} 1 \\ e^{iF_0(x_{0+})/2} \end{array} \right) e^{ik_y x + f_{0+}^+ \sigma} e^{i\pi/4},
\]

(7a)

\[
\Psi^\text{out}_+ = \frac{1}{\sqrt{2 \nu_{y+}(x)}} \left( \begin{array}{c} 1 \\ e^{iF_0(x_{0+})/2} \end{array} \right) e^{ik_y x + f_{0+}^+ \sigma} e^{i\pi/4}.
\]

(7b)

On the trajectory described by Eqs. (7), the electron spin follows the orbital motion and is always perpendicular to the momentum \( \vec{k} = (k_x(x), k_y) \), as shown in Fig. 3(c).

To conclude this section, we note that the trajectory (a) in Fig. 3 can be easily described within the quasiclassical approach as the spin and the coordinate \( x \) can be separated. In the case of the trajectory (b), the incoming + wave creates two outgoing waves. This case should be treated separately introducing slow conversion of waves in the barrier region. Another solvable limit is \( a \to 0 \). In this case, the barrier becomes rectangular and we can analytically solve the problem of reflection using the plane waves and exponential functions.3

V. NUMERICAL RESULTS

Now we present numerical results that support the above quasiclassical consideration. The barrier potential \( U(x) \) has been specified above. Again we use the typical parameters of InSb quantum wells.8 Experimentally, the conventional methods to fabricate lateral barriers are etching of surface or deposition of a metallic gate; with the above methods, the typical lateral dimensions for the barrier potential cannot be made too short. Typically, they are in the sub-\( \mu \)m range. Figure 4 shows calculated reflection coefficients for the barriers with different widths \( a \). The reflection coefficients are defined in the following way:

\[
R_{+-} = \left| \frac{A_{+-}}{A_{\text{in}}} \right|^2, \quad R_{-+} = \cos \theta_{++} \frac{A_{+-}}{A_{\text{in}}} \cos \theta_{--} \frac{A_{--}}{A_{\text{in}}}.
\]

where \( A_{+-} \) are the coefficients of the wave function outside of the barrier, in the right-hand side of the system. The above coefficients satisfy the conservation-of-charge law: \( R_{++} + R_{--} = 1 \) and \( R_{+-} + R_{-+} = 1 \). We obtained these equations considering the \( x \) component of the current operator. It is seen from Fig. 4(a) that, for a moderate SOI in the InSb system, the reflection coefficients weakly depend on the barrier width and the spin conversion remains very strong even for very smooth lateral barriers. For the barrier \( a = 100 \) Å, the reflection coefficients are very close to those of the hard reflecting wall. For the hard-wall barrier, we can use a simple geometrical consideration for the spin conversion at the barrier and obtain \( R_{--} = R_{++} = 1 - \cos(2\theta)/2 \) and \( R_{+-} = R_{-+} = 1 + \cos(2\theta)/2 \).3 These simple equations are valid if \( \sqrt{\delta} \ll 1 \) and \( \pi/2 - \theta \gg \sqrt{\delta} \). It is also seen from Fig. 4(a) that the off-diagonal coefficients and the spin-conversion effect decrease with increasing the barrier width \( a \). It is expected from the quasiclassical theory. The off-diagonal coefficients \( R_{-+-} \) and \( R_{+-+} \) do not decrease much with increasing \( a \) because

FIG. 4. Diagonal and off-diagonal reflection coefficients as a function of the incident angle. The barrier heights are 150 meV (a) and 70 meV (b). The 2D density is \( 2.1 \times 10^{11} \text{ cm}^{-2} \) and \( E_F = 35 \text{ meV} \).
the parameter of the quasiclassical theory \( \eta = (eF_0\lambda_F)/\Delta_{SO} = (U_0\lambda_F/a)/\Delta_{SO} \) remains large even for the longest barrier. For the longest barrier width of 10 000 Å and \( U_0 = 150 \text{ meV}, \eta \approx 3.6 \). This reflects also a relatively small strength of SOI in the conduction band of InSb. Suppression of spin-flip processes for reflection from the smoother potential can be seen in Fig. 4(b), which shows the reflection coefficients for the barrier of \( U_0 = 70 \text{ meV} \) \( (\eta \approx 1.8) \). Figure 5 demonstrates \( R_{\alpha\alpha'} \) for different parameters \( \Delta_{SO} \) and for a sharp barrier with \( a = 100 \text{ Å} \). The very strong effect coming from \( \Delta_{SO} \) is seen in reflection of the wave \( \Psi_{in} \), this effect originates from the kinematics of scattering. As for the reflection coefficients \( R_{++} \) and \( R_{--} \), the effect of the SOI constant \( \Delta_{SO} \) is not strong; with increasing \( \Delta_{SO} \), \( R_{++} \) slightly increases as it is expected from the semiclassical theory. Numerically, the quasiclassical regime of reflection can be obtained for strong SOIs and smooth barriers. In Fig. 6, we show the functions \( R_{\alpha\alpha'}(a) \) for the case of \( \Delta_{SO} = 10^5 \text{ meV cm} \), which can exist in other narrow-band semiconductors. It is seen that the off-diagonal reflection coefficients strongly decrease with increasing the width \( a \), whereas the diagonal coefficients approach unity. For \( a = 10 000 \text{ Å} \), the parameter \( \eta = 0.36 < 1 \) and the spin-orbit motion of electron becomes adiabatic.

VI. DISCUSSION

An observation of the predicted multibeam reflection depends on two factors. On one hand, one needs a sufficiently strong SOI, which would result in strong spatial separation of beams with different spin polarizations [Figs. 1 and 2(c)]. On the other hand, typical lateral dimensions of etched mesoscopic structures are in the sub-μm range and, if the SOI is very strong, the spin-conversion efficiencies \( (R_{+++} \) and \( R_{---} \)) can become small. Here we found that the InSb quantum wells would be a suitable system to observe this effect. InSb quantum wells have the moderate SOI, which results in the spin-dependent angular deviations, \( |\theta_{+++} - \theta| \) and \( |\theta_{---} - \theta| \), of order of \( 10^9 \). At the same time, the off-diagonal reflection coefficients for relatively smooth barriers remain large. Another suitable system to observe spin-dependent reflection can be a 2D hole gas in GaAs quantum wells where the SOI is quite strong due to the mixing between heavy and light holes.

To conclude, we have studied the physical properties of reflection of electrons from a lateral barrier in a narrow-gap semiconductor. Using the typical parameters of InSb quantum wells, we show that the effect of multibeam reflection remains strong for realistic lithographical barriers. The spin-dependent reflection described in this paper can be used for spin-filtering devices based on ballistic nanostructures.

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