Three-Body Scattering Without Partial Waves

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3B Scattering

Elastic Process  sum of Projectile-Target interactions

\[ U|\phi\rangle = PG_0^{-1}|\phi\rangle + PT|\phi\rangle \]

Breakup process:  sum of interaction within “all” pairs

\[ U_0^{(1)}|\phi\rangle = \left(1 + P\right)T|\phi\rangle \]

kinematically allowed if

\[ E_{th} > \frac{3}{2}E_d \]
3B Transition Amplitude

\[ T \phi = tP \phi + tG_0 PT \phi \]

The Faddeev Equation in momentum space by using Jacobi Variables

\[
\langle p q | \hat{T} | \varphi_d q_0 \rangle = \varphi_d(q + \frac{1}{2}q_0) \hat{t}_s(p, \frac{1}{2}q + q_0, E - \frac{3}{4m}q'^2) + \int d^3q'' \frac{\hat{t}_s(p, \frac{1}{2}q + q'', E - \frac{3}{4m}q'^2)}{E - \frac{1}{m}(q'^2 + q''^2 + q \cdot q'')} \langle q + \frac{1}{2}q'', q'' | \hat{T} | \varphi_d q_0 \rangle
\]

Traditionally solved by Partial Wave Decomposition scheme

At low energy (<250MeV): practically works well

At high energy (>250MeV): more and more PWs are needed

algebraically and computationally difficult
Three Dimensional Calculation:

Solving Faddeev equation directly in vector momentum space

Applicable to high energy case in order to explore few-body dynamics at short distance, especially the inclusion of 3BF and relativity.

\[ p = |\mathbf{p}|, \quad q = |\mathbf{q}|, \]

\[ x_p = \hat{\mathbf{p}} \cdot \hat{\mathbf{q}}_0, \quad x_q = \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}_0 \]

\[ x_{pq}^{q_0} = (\hat{\mathbf{q}}_0 \times \hat{\mathbf{q}}) \cdot (\hat{\mathbf{q}}_0 \times \hat{\mathbf{p}}) \]

\( q \) system : \( \mathbf{z} \parallel \mathbf{q} \)

\( q_0 \) system : \( \mathbf{z} \parallel \mathbf{q}_0 \)

Variables invariant under rotation:

freedom to choose coordinate system for numerical convenience
Angular Relations in q system

\[ \varphi_{q_0} = \text{anything} \]

\[ y_{pq} = \hat{p} \cdot \hat{q} = x_p x_q + \sqrt{1 - x_p^2} \sqrt{1 - x_q^2} x_{pq}^{q_0} \]

\[ \cos(\varphi_p - \varphi_{q_0}) = \frac{x_p - y_{pq} x_q}{\sqrt{1 - y_{pq}^2} \sqrt{1 - x_q^2}} \]

\[ \sin(\varphi_p - \varphi_{q_0}) = \sqrt{1 - \cos^2(\varphi_p - \varphi_{q_0})} \text{ or } -\sqrt{1 - \cos^2(\varphi_p - \varphi_{q_0})} \]

\[ \cos \varphi_p = \cos \varphi_{q_0} \cos(\varphi_p - \varphi_{q_0}) - \sin \varphi_{q_0} \sin(\varphi_p - \varphi_{q_0}) \]

\[ \sin \varphi_p = \sin \varphi_{q_0} \cos(\varphi_p - \varphi_{q_0}) + \cos \varphi_{q_0} \sin(\varphi_p - \varphi_{q_0}) \]

\[ y_{q_0 q''} = \hat{q}_0 \cdot q'' = x_q x'' + \sqrt{1 - x_q^2} \sqrt{1 - x''^2} \cos(\varphi_{q_0} - \varphi'') \]

\[ y_{pq''} = \hat{p} \cdot q'' = y_{pq} x'' + \sqrt{1 - x''^2} \sqrt{1 - y_{pq}^2} \cos(\varphi_p - \varphi'') \]

\[ x_{\Pi_p} = \hat{\Pi}_p \cdot q_0 = \frac{q x_q + \frac{1}{2} q'' y_{q_0 q''}}{\sqrt{q^2 + \frac{1}{4} q''^2 + q q'' x''}} \]

\[ x_{\Pi_q} = \hat{\Pi}_q \cdot q_0 = y_{q_0 q''}. \]

\[ y_{qq''} = \hat{q} \cdot \hat{q}'' = x'' \]

Working in q system simplified

the structure of moving pole
3D integral equation with 5 variables

\[ \langle p, x_p, x_{pq}^0, x_q, q | \hat{T} | q_0 \varphi_d \rangle \]

\[ = \varphi_d \left( \sqrt{q^2 + \frac{1}{4} q_0^2 + q q_0 x_q} \right) \]

\[ \times \hat{t}_s \left( p, \sqrt{\frac{1}{4} q^2 + q_0^2 + q q_0 x_q}, \frac{1}{2} q y_{pq} + q_0 x_p}{\sqrt{1/4 q^2 + q_0^2 + q q_0 x_q}}; E - \frac{3}{4m} q^2 \right) \]

\[ + \int_0^\infty dq'' q''^2 \int_{-1}^1 dx'' \int_0^{2\pi} d\varphi'' \frac{1}{E - \frac{1}{m} (q^2 + q q'' x'' + q''^2) + i\varepsilon} \]

\[ \times \hat{t}_s \left( p, \sqrt{\frac{1}{4} q''^2 + q'' x''}, \frac{1}{2} q y_{pq} + q'' y_{pq''}{\sqrt{1/4 q''^2 + q'' x''}}; E - \frac{3}{4m} q''^2 \right) \]

\[ \times \left( \sqrt{q^2 + \frac{1}{4} q''^2 + q q'' x''}, -\frac{q x_q + \frac{1}{2} q'' y_{q0 q'}^{q''}}{\sqrt{q^2 + \frac{1}{4} q''^2 + q q'' x''}}, \frac{q x_q + \frac{1}{2} q'' y_{q0 q'}^{q''}}{\sqrt{1-x^2_{\pi p}}} \right) \]

\[ \times \left( \sqrt{1-x^2_{\pi q}}, y_{q0 q''}, q'' | \hat{T} | q_0 \varphi_d \right) \]

\[ E - \frac{3}{4m} q''^2 - E_d + i\varepsilon \]

The position of the pole in three-body propagator depends on \( q, q'' \) and \( x'' \)

Moving singularity

Fixed “deuteron” pole
Moving Singularity In $x^{'''}$ Integration

\[
\int_0^\infty dq'' \int_{-1}^{+1} dx'' \frac{F(q'',x'')}{(x_0 - x'' + i\varepsilon)(q_0^2 - q''^2 + i\varepsilon)}
\]

\[
= \int_0^{q_{\text{max}}} dq'' \int_{-1}^{+1} dx'' \frac{1}{(q_0^2 - q''^2)} \frac{F(q'',x'')}{(x_0 - x'' + i\varepsilon)} + \int_{q_{\text{max}}}^{\infty} dq'' \int_{-1}^{+1} dx'' \frac{1}{(x_0 - x'')(q_0^2 - q''^2 + i\varepsilon)} \frac{F(q'',x'')}{}
\]

\[
x_0 = \frac{mE - q^2 - q''}{qq'''}
\]

\[
q_+ = \frac{q}{2} + \sqrt{Q_0^2 - \frac{3}{4}q^2}
\]

\[
q_- = \frac{q}{2} - \sqrt{Q_0^2 - \frac{3}{4}q^2}
\]

\[
Q_0 = \sqrt{mE}, \quad q_{\text{max}} = \sqrt{\frac{4m}{3}E}, \quad q_0 = \sqrt{\frac{4m}{3}(E - E_d)}, \quad Q_0 < q_{\text{max}} < q_0.
\]
Logarithmic Singularity $q'$ Integration

\[ \int_{0}^{q_{\text{max}}} dq'' \int_{-1}^{1} dx'' \frac{F(q'',x'')}{x'' - x_0 - i\varepsilon} = \int_{0}^{q_{\text{max}}} dq'' F(p'',x_0) \int_{-1}^{1} dx \frac{1}{x'' - x_0 - i\varepsilon} + \int_{0}^{q_{\text{max}}} dq'' \int_{-1}^{1} dx \frac{F(p'',x) - F(p'',x_0)}{x'' - x_0} \]

\[ \ln \left| \frac{1 + x_0}{1 - x_0} \right| + i\pi \Theta (1 - |x_0|) \]

\[ \ln \left| \frac{1 + x_0}{1 - x_0} \right| = \left( -\frac{q_-}{q_-} \ln |q'' + |q_-|| - \ln |q'' + |q_+|| \right) \]

\[ \quad + \left( +\frac{q_-}{q_-} \ln |q'' - |q_-|| + \ln |q'' - |q_+|| \right) \]

\[ \int_{q_-}^{q_+} dq'' F(q'',x_0) \ln |q'' - |q_\pm|| \]

The integral with logarithmic singularity carried out by Spline based method. More efficient!

$\mathcal{R}(q',x_0)$ approximated by cubic splines and the integration carried out analytically
The Self Consistency Check

Solution of Faddeev equation and coordinate independence

Results from reinseration

\[ \sigma_{q_0} = 2561.6736 \text{mb} \]
\[ \sigma_q = 2561.5359 \text{mb} \]

Results from Pade summation

\[ E_{lab} = 3.0 \text{MeV} \]

\[ T|\phi\rangle = tP|\phi\rangle + tG_0 PT|\phi\rangle \]

\[ \sigma_{q_0} = 2561.7364 \text{mb} \]
\[ \sigma_q = 2561.1384 \text{mb} \]
The Unitarity non trivial, nonlinear relation

\[
\langle \phi | U | \phi' \rangle^* - \langle \phi' | U | \phi \rangle = \int d\mathbf{q} \langle \phi_q | U | \phi' \rangle^* 2\pi i \delta(E - E_q) \langle \phi_q | U | \phi \rangle + \frac{1}{3} \int d\mathbf{p} d\mathbf{q} \langle \phi_0 | U_0 | \phi' \rangle^* 2\pi i \delta(E - E_{pq}) \langle \phi_0 | U_0 | \phi \rangle
\]

\[
- \frac{4m(2\pi)^3}{3q_0} \text{Im} \langle q_0, 1, \varphi_d | U | q_0 \varphi_d \rangle = \sigma_{tot}
\]

\[
E_{lab} = 10\text{MeV}
\]

\[
- \frac{4m(2\pi)^2}{3q_0} \text{Im} \langle q_0 \varphi_d | U | q_0 \varphi_d \rangle = 1904.98\text{mb}
\]

\[
\sigma_{tot} = \sigma_{el} + \sigma_{br} = 1820.77 + 73.75 = 1894.52\text{mb}
\]

\[
\Rightarrow 0.5\%
\]

\[
E_{lab} = 500\text{MeV}
\]

\[
- \frac{4m(2\pi)^2}{3q_0} \text{Im} \langle q_0 \varphi_d | U | q_0 \varphi_d \rangle = 106.87\text{mb}
\]

\[
\sigma_{tot} = \sigma_{el} + \sigma_{br} = 63.78 + 48.51 = 112.28\text{mb}
\]

\[
\Rightarrow 5.1\%
\]

Can be improved with finer grid
Rescattering Effects

$E_{\text{lab}} = 200$ MeV

\[ T|\phi\rangle = tP|\phi\rangle + tG_0 PT|\phi\rangle \]

1. Forward: $10^3$ mb
2. Backward: $10^0$-$10^1$ mb
3. Adding term by term: monotonically toward convergence
4. Rescattering terms are important
Rescattering Effects

$E_{\text{lab}} = 500\text{MeV}$

\[ T|\phi\rangle = tP|\phi\rangle + tG_0 PT|\phi\rangle \]

1. Forward: $10^2$-$10^3$ mb
2. Backward: $10^{-2}$ – $10^{-1}$mb
3. Adding term by term:
   alternatively toward convergence
4. Rescattering terms are still important
Summary & Outlook

• The property of rescattering effects has strong energy dependence.

• Study convergence of multiple scattering series through breakup observables at different energy regions and kinematic configurations.

• Check “typical” “high energy” approximations (Glauber)
Computation Model

Multileveled by MPI+OpenMP

Well suited on cluster machine with ideal scalability and high performance.