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Relativistic Dynamics for the $\pi - N$ System – p.2/32
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- Bethe-Salpeter a lâ K-matrix.

\[ T = K + K \delta^{(4)} T \quad K = V + V G^P \quad K \approx V , \]

\[ \delta^{(4)} = \text{on-mass shell. No off-shell solutions.} \]
**π — N Bethe-Salpeter Equation**

The operator equation in general is:

\[ T(s) = V(s) + V(s) G_{\pi N}(s) T(s) \]

Here

\[ V(s) = \text{Sum of all two-particle irreducible diagrams} \]

\[ G_{\pi N}(s) = d_\pi d_N = \text{The } \pi N \text{ propagator} \]

\[ d_\pi(q) = (q^2 - \mu_0 - \Pi(q))^{-1}, \quad \Pi(q) = \text{pion dressing} \]

\[ d_N(p) = (p^2 - m_0 - \Sigma(p))^{-1}, \quad \Sigma(p) = \text{nucleon dressing} \]

As it stands it has never been solved, even for the simplest Lagrangian.
1. \( V(s) = \text{Sum of lowest order tree diagrams for a given Lagrangian, i.e we have a potential that could have some of the symmetry of QCD, e.g. chiral symmetry.} \)
Approximations

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   Replace \( \mu_0 \) and \( m_0 \) by the physical masses
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3. Some of the higher unitarity cuts are included through $V$ and the pinching of $V$ with $G_{\pi N}$ – more later.

   This is known as the Ladder Bethe-Salpeter Equation, which has the problem that for equal mass initial state does not have the “one-body” limit.
Unitarity structure of the Amplitude

The integral part of the BS equation is of the general form

\[ \int d^2 q'' V_\ell(q, q''; s) G(q''; s) T_\ell(q'', q'; s) , \]

There are basically three ways one can generate unitarity cuts in the BS amplitude. These involve the pinching of the \( q_0 \) contour by:

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Let me examine some of these singularities just to illustrate the point.
Pinching of contour by poles $G(q''; s)$

In the figure we have the poles of $G(q'', s)$ for two equal mass scalar particles in the $q''_0$ plane. The arrow indicates the direction the poles move as $\sqrt{s}$ increases.

The pinching of the $q''_0$ integration path give rise to a branch point at

$$\sqrt{s} = \pm 2m$$

Note: The resultant amplitude has a branch points at $s = 4m^2$. 

```
Pinching in $V(q, q''; s) G(q''; s)$

The potential for one particle exchange with mass $\mu$ has logarithmic branch points after partial wave expansion. These are illustrated in the Figure, and their position does not depend on $s$. For $\sqrt{s} > 0$ The pinching of (2) & (8) give a branch point at
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The pinching of (3) & (6) give a branch point at

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In a similar manner we can find two branch points for $\sqrt{s} < 0$. 
Pinching of $G(q''; s) \ T(q'', q'''; s)$

The off-mass shell amplitude branch points can, with the help of the propagator poles, pinch the integration contour generating branch points in the on-shell amplitude. See Figure.
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The pinching of (2) & (10) give a branch point in the on-shell amplitude at $\sqrt{s} = 2m + \mu$. 

---

**Figure:**

- Branch point at $\sqrt{s} = 2m + \mu$.
- The on-shell amplitude $T(q'', q'''; s)$ with thresholds at $s = (2m + \mu)^2$.
The off-mass shell amplitude branch points can, with the help of the propagator poles, pinch the integration contour generating branch points in the on-shell amplitude. See Figure.

The pinching of (2) & (10) give a branch point in the on-shell amplitude at $\sqrt{s} = 2m + \mu$. This is the three-body unitarity cut.
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The pinching of the branch point in $T$ with that of $V$ generates an off-mass shell branch point in $T$. 

\[ G(q''; s) \ T(q'', q'''; s) \]
Pinching of $G(q''; s) \, T(q'', q'''; s)$

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The pinching of the branch point in $T$ with that of $V$ generates an off-mass shell branch point in $T$. This in turn will pinch the contour with $G$ to give the four-body unitarity cut, $\Rightarrow$ BS amplitude has thresholds at $s = (2m + n\mu)^2 \ , \ n = 1, 2, \cdots$. 

\[ \sqrt{s} > 0 \]
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The $\pi N$ Potential

The following tree diagrams have been included in $V$

![Tree diagrams](image)

All $\pi$ coupling are taken to be derivative coupling

$\Rightarrow$ Lagrangian consistent with chiral symmetry.

$\Rightarrow$ Born amplitude chirally symmetric.
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The Rarita-Schwinger propagator:

\[
P_{\mu\nu}^{RS}(p) = \frac{p + m_\Delta}{p^2 - m_\Delta^2} \mathcal{P}^{3/2} + \frac{2(p + m_\Delta)}{3m_\Delta^2} \mathcal{P}^{1/2}_{22} + \frac{1}{\sqrt{3m_\Delta}} \left( \mathcal{P}^{1/2}_{12} + \mathcal{P}^{1/2}_{21} \right)
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The Propagator

- The Rarita-Schwinger propagator:

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\[ + \frac{1}{\sqrt{3m_\Delta}} \left( P_{12}^{1/2} + P_{21}^{1/2} \right) \]

- The Williams propagator:

\[ P_{\mu\nu}^{W}(p) = \frac{\not{p} + m_\Delta}{p^2 - m^2_\Delta} P^{3/2} \]

Relativistic Dynamics for the $\pi - N$ System – p.12/32
The Δ Propagator

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- The Williams propagator:

\[
P_{\mu\nu}^{W}(p) = \frac{p + m_\Delta}{p^2 - m_\Delta^2} P^{3/2}
\]

- The Pascalutsa propagator:

\[
P_{\mu\nu}^{Pas}(p) = \frac{p + m_\Delta}{p^2 - m_\Delta^2} p^2 P^{3/2}
\]
The $\Delta$ propagators

As a first fit we compare the results for the two different $\Delta$ propagators.

— Rarita-Schwinger (RS)
- - Pascalutsa (Pas)

Note 1: The fit to the data is very good especially for the Rarita-Schwinger propagator.

Note 2: The difference in fit might be a result of imperfect search.
**Cut-Off – Form Factors**

To solve the Bethe-Salpeter Eq., we need some form of cut-off in the integral part of the equation. We introduce one of two form factors.

- **Type I:** A function of the momentum square of all three legs:

  \[ f_{\alpha\beta\gamma}(p^2_\alpha, p^2_\beta, p^2_\gamma) = f_\alpha(p^2_\alpha) f_\beta(p^2_\beta) f_\gamma(p^2_\gamma) \]

  where

  \[ f_\alpha(p^2_\alpha) = \left( \frac{\Lambda^2_\alpha - m^2_\alpha}{\Lambda^2_\alpha - p^2_\alpha} \right)^{n_\alpha} \]
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  \]

- **Type II:** A function of the momentum square of the \( \pi \) only

  \[
  f_{\alpha\beta\gamma}(p_\alpha^2, p_\beta^2, p_\gamma^2) = f_\pi(p_\pi^2)
  \]
The Role of the Form Factors

Here we compare the phase shifts for the two different form factors:
— Type I (product)
- - Type II with \( n = 4 \)
\[
f_{\pi}(p^2) = \left( \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - p^2} \right)^n
\]

Note: Type II form factor has less parameters and is equivalent to Pauli-Villars regularization.
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How High in Energy can we get?

With parameters adjusted to fit data at lower energies the phase shifts at higher energies.

— Type I form factor RS
- - - Type I form factor Pas
-- Type II form factor

\[ n = 4 \text{ RS} \]

\[ \sqrt{s} < 2(m_{\pi} + m_{\sigma}) \approx 1.6 \text{ GeV}, \text{ due to pinching singularities.} \]

For \( \sqrt{s} > 2(m_{\pi} + m_{\sigma}) \) Wick rotation is a problem.

Note: The main discrepancy is for resonant phase shifts, e.g. \( S_{11} \).
How about the 3-D reductions?

The Bethe-Salpeter equation is a 4-D integral equation given as

\[ T(q', q; s) = V(q', q; s) - \frac{i}{(2\pi)^4} \int d^4q'' \ V(q', q''; s) \ G_{\pi N}(q''; s) \ T(q'', q; s) , \]

where \( G_{\pi N} \) is the \( \pi N \) propagator and \( V \) is the sum of all the tree diagrams.
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We would like to replace this equation by a 3-D equation of the form

\[ \bar{T}(\bar{q}', \bar{q}; E) = \bar{V}(\bar{q}', \bar{q}; E) + \int d^3 q'' \bar{V}(\bar{q}', \bar{q}''; E) \bar{G}_{\pi N}(\bar{q}''; E) \bar{T}(\bar{q}'', \bar{q}; E) . \]
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There is an infinite number of ways to achieving this. I will consider only five that differ in the unitarity structure of the final amplitude.
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Equal-Time (ET) — Klein (1953)

Preserve two-body & some three-body unitarity cuts:

- **Two-body unitarity** $G_{\pi N} \Rightarrow \langle G_{\pi N} \rangle \equiv \tilde{G}_{\pi N}$ where

  $$\langle A \rangle = \int dq_0 dq'_0 A(q_0, \bar{q}; q'_0, \bar{q}'; s)$$

- **Three-body unitarity** — pinch contour between $G_{\pi N}$ and $T$ with $T \approx VG_{\pi N}$, i.e. $G_{\pi N}T \Rightarrow \langle G_{\pi N}VG_{\pi N} \rangle$, or

  $$V \Rightarrow \langle G_{\pi N} \rangle^{-1} \langle G_{\pi N} V G_{\pi N} \rangle \langle G_{\pi N} \rangle^{-1} \equiv \tilde{V}.$$
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  \[
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  \]

This gives us thresholds at

\[
\sqrt{s} = m_N + \mu_\pi + \mu_H \quad \sqrt{s} = 2m_N + m_H \quad \sqrt{s} = 2\mu_\pi + m_H
\]
Cohen (C) approximation — (1970)

In this case two-body & some four-body unitarity are included by taking

\[ T(q'_0, q'; \bar{q}_0, \bar{q}; s) \approx \tilde{T}(\bar{q}_0, q'; \bar{q}_0, \bar{q}; s), \]

where \( \bar{q}_0 \) is the on-mass-shell value. This allows us to evaluate

\[ \int dq''_0 \, V(q, q''; s) \, G_{\pi N}(q'', s) \]

In this case the \( t \)-channel and propagator poles give branch point at

\[ \sqrt{s} = m_N + \mu_\pi + 2\mu_H \quad \text{with} \quad H = \rho, \sigma \]

while the \( u \)-channel and propagator poles give thresholds at

\[ \sqrt{s} = 3m_N - \mu_\pi + 2m_H \quad \sqrt{s} = -m_N + 3\mu_\pi + 2m_H \quad H = N, \Delta \]
Preserve two-body unitarity & charge conjugation in the $\pi N$ propagator

$$V(q'_0, q'; q_0, q; s) \approx \bar{V}(\bar{q}_0, q'; \bar{q}_0, q; s),$$

$\bar{q}_0$ fixed to on-shell value. As a result the contour in pinched by the poles of $G_{\pi N}$ and you have a branch point at $\sqrt{s} = \pm (m_N + m_\pi)$. The amplitude is a function of $s$. 

**Instantaneous — Tjon & Pascalutsa**
From noon to six o'clock we ran thirty miles to the northward skirting a sandy shore at the distance of five, and then to eight miles; the depth was then 5 fathoms, and we dropped the anchor up on a bottom of sand, mixed with pieces of dead coral. From hence and from some other cross bearings, to be 34° 59' south and 138° 42' east. No land was visible so far to the north as where the trees appeared above the horizon, which showed the coast to to very low, and our soundings were The situation of Mount Lofty was found fast decreasing.

**Blankenbecler-Sugar**

- **Two-body unitarity:** There are an infinity of equations

\[
\tilde{G}_{\pi N}(q, P) = \frac{1}{2\pi i} \int_0^\infty \frac{ds' f(s', s)}{s' - s - i\epsilon} \Delta [G_{\pi N}]
\]

where \( f(s', s) \) is arbitrary, with the condition that \( f(s, s) = 1 \). We will consider Blankenbecler-Sugar equation (BbS)

\[
f(s', s) = 1.
\]
Two-body unitarity: There are an infinity of equations

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We will consider Blankenbecler-Sugar equation (BbS) \( f(s', s) = 1 \).

For the Cooper-Jennings (CJ) approximation \( f(s', s) \) is chosen such that we get equal contribution from positive and negative energies parts of the \( \pi N \) propagator.
Comparison of 3-D Eqs. — $S$-wave

Figure: The phase shifts for type I (a) and type II (b) form factors

Note: The more of the analytic structure of the BS equation you keep, the better the agreement with the BS results.
From noon to six o'clock we ran thirty miles to the northward skirting a sandy shore at the distance of five, and then to eight miles; the depth was then 5 fathoms, and we dropped the anchor up on a bottom of sand, mixed with pieces of dead coral.

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Comparison of 3-D Eqs. — $P_{11}$

Figure: The phase shifts for type I (a) and type II (b) form factors

Note: We have used the parameters of the Bethe-Salpeter fit to calculate all phase shifts for the 3-D equations.
From noon to six o'clock we ran thirty miles to the northward skirting a sandy shore at the distance of five, and then to eight miles; the depth was then 5 fathoms, and we dropped the anchor up on a bottom of sand, mixed with pieces of dead coral. From hence and from some other cross bearings, to be 34°59′ south and 138°42′ east. No land was visible so far to the north as where the trees appeared above the horizon, which showed the coast to be very low, and our soundings were The situation of Mount Lofty was found fast decreasing.

Comparison of 3-D Eqs. — $P_{33}$

Figure: The phase shifts for type I (a) and type II (b) form factors

Note: We have used the parameters of the Bethe-Salpeter fit to calculate all phase shifts for the 3-D equations.
Only on-mass-shell amplitude are included, \( i.e. \)

\[
G_{\pi N}(q'', s) \rightarrow \langle G_{\pi N} \rangle \rightarrow -i\pi\rho(E)\delta^{(4)}(q - q_{on})
\]

- Off-shell effects are not present. We only need on-mass-shell amplitude.
- The potential or \( K \)-matrix can be derived from a chiral perturbation theory.
- Equation are algebraic, therefore we can generalize to coupled channels.
- \( \rho(E) \) is the density of state, or \( \langle G_{\pi N} \rangle \).

The results reported here are for \( \rho(E) \) density of states.
Comparison for $K$-matrix — $S$-wave

Figure: The phase shifts for type I (a) and type II (b) form factors

Note: We have used the parameters of the Bethe-Salpeter fit to calculate all phase shifts for $K$-matrix approximation.
Comparison for $K$-matrix — $P_{11}$

Figure: The phase shifts for type I (a) and type II (b) form factors

Note: We have used the parameters of the Bethe-Salpeter fit to calculate all phase shifts for $K$-matrix approximation.
Comparison for $K$-matrix — $P_{33}$

Figure: The phase shifts for type I (a) and type II (b) form factors

Note: We have used the parameters of the Bethe-Salpeter fit to calculate all phase shifts for $K$-matrix approximation.
How good is the \( K \)-matrix

- For partial waves dominated by resonance \( e.g. P_{33} \) — very good.
- For partial waves with no resonance \( e.g. S_{31} \) — it can be good.
- For partial waves with a mix \( e.g. P_{11} \) — there are problems.
Conclusions

- One can use the Bethe-Salpeter equation and fit the data over a wide energy region with some success.
- No ambiguity about the starting equation other than truncation of $V$.
- Resonances and their dressing and renormalization are included consistently, *i.e.*

$$T(s) = T_{\text{pole}}(s) + T_{\text{non-pole}}(s)$$

- The gauging of the final equation well defined — form factors are a problem.
Conclusions — cont.

Advantages:

- The unitarity structure of the covariant formulation is preserved.
- The off-mass-shell amplitude can be used to calculate amplitudes for:
  - Use in evaluation of \( \gamma N \rightarrow \pi N \)
  - Examine low energy theorem
- Make use of chiral perturbation theory for the potential as in \( NN \).
From noon to six o'clock we ran thirty miles to the northward skirting a sandy shore at the distance of five, and then to eight miles; the depth was then 5 fathoms, and we dropped the anchor up on a bottom of sand, mixed with pieces of dead coral.

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Conclusions — cont.

Disadvantage

- Need for computing power to fit the data — time is on our side.
- Need for off-shell information — Not a complete solution to field theory.
- Ladder approximation does not have one-body limit.
- Crossing is not included.