Compton scattering on the nucleon in the 'Dressed K-matrix' approach

causality in a coupled channels formalism

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Content

- Motivation
  - Coupled Channels; Covariant; Gauge invariant; Unitary; Causal; Crossing symmetric

- Dressing in a K-matrix approach
  - Formalism; Vertex and self-energy functions

- Results
  - Pion photoproduction; Compton scattering; Polarizabilities; Sumrules

- Summary

Description pion and photon induced reactions on nucleon very low & intermediate energies $\approx 1$ GeV
Coupled Channels formalism

Simplest: K-matrix approach, \[ T = \frac{K}{1 - iK} \]

\( T \) and \( K \) are matrices in channel space

- Kernel = sum of tree-level diagrams
- Include full (non-strange) resonance spectrum (below 2 GeV)
- Coupled channels \[ \begin{align*}
\gamma + p & \rightarrow \gamma + p \\
\pi + p & \rightarrow \pi + p
\end{align*} \]
- Unitary below \( 2\pi \) production threshold;
  \( 2\pi \) In-elasticities via width in resonance propagators
- Gauge invariant (minimal substitution)
- Relativistic
- Crossing symmetry (real \( k \))
- Good fit to \( \pi + p \rightarrow \pi + p \), \( \gamma + p \rightarrow \pi + p \), and \( \gamma + p \rightarrow \gamma + p \) simultaneously
- Also applied to virtual Compton scattering
s-u Crossing Symmetry

- symmetry under

Obeyed in K-matrix formalism
(Provided K is cross. sym.)

- Proof

 Included through formalism

Corresponds to nucleon decaying into nucleon + 2 other physical particles.

Impossible !!!

crossed diagrams vanish !!!!

Observable:
expansion coefficients Compton multipoles
K-matrix approach

Summary
Convenient for obeying unitarity since on-shell rescattering (Im loops corrections) taken into account in T

simple example:

\[ K = \quad \]

\[ T = \quad + \quad \text{Im} \quad + \quad \text{Im} \quad + \quad \text{Im} \quad + \ldots \]

\[ T = K + K \text{i} K + K \text{i} K \text{i} K + \ldots \]

\[ = \frac{K}{1 - \text{i} K} \]

Major problem: Off-shell rescattering (Re loops) neglected

Consequence: Causality violated!

Causality ~ Analyticity ~ Dispersion relations

<table>
<thead>
<tr>
<th></th>
<th>K-matr</th>
<th>( \chi_{PT} )</th>
<th>Dressed K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unitary</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>Covariance</td>
<td>OK</td>
<td>??</td>
<td>OK</td>
</tr>
<tr>
<td>Gauge Invariance</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>Crossing</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>Causality</td>
<td>NO</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>Low ( E_\gamma )</td>
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<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>( \Delta )-resonance</td>
<td>OK</td>
<td>NO</td>
<td>OK</td>
</tr>
<tr>
<td>Higher Energ.</td>
<td>OK</td>
<td>NO</td>
<td>OK</td>
</tr>
</tbody>
</table>
Compton scattering on the proton

Polarizabilities: \((\text{PRC58}(1998)1098)) \ [10^{-4} \text{ fm}^4]\)
\[\alpha = 5.6 \quad \text{(data: 11.9)}\]
\[\beta = 2.6 \quad \text{(data: 1.9)}\]
The "Dressed K-matrix" approach


Analyticity of amplitude is restored
(approximately)
by using the 'Dressed K-matrix'

Analyticity important for:

- Amplitude near particle threshold
- Sum rules

Basic idea:

Construct real vertex and self-energy functions from Hilbert Transform of cut-loop contributions
**Simple example**

Scattering amplitude up to 1 loop
(oversimplified example)

\[K = \quad \quad \]

\[T = \quad + \quad + 2, 3, ... \text{ Im loops} \]

\[\quad = \quad + \quad + 2, 3, ... \text{ Re loops} \]

\[\quad = \quad + \quad + 2, 3, ... \text{ Re loops} \]

\[T^{(1)} = \quad + \left[ \quad + \quad \right] + \ldots \]

**T includes Im and Re parts**

Real parts of loop corrections are taken into account as vertex and self-energy functions
Dressing the vertex and propagators

\[ \text{Off-shell} \quad \begin{array}{c}
\text{On-shell} \\
\text{Bare } \pi NN \text{ vertex}
\end{array} \]

\[ + \int \left[ \begin{array}{c}
\text{Disp.} \\
\text{Disp.}
\end{array} \right] \]

\[ \text{Free } N \text{ propagator} \]

Solved by iteration, at each iteration step:

- Cutting rules (unitarity) \( \Rightarrow \text{Im } \Gamma, \text{Im } \Sigma \)
i.e. \( \text{Im} \) parts \( G_s(p^2), G_v(p^2), A(p^2), B(p^2) \)

- Dispersion relations (causality) \( \Rightarrow \text{Re } \Gamma, \text{Re } \Sigma \)
i.e. \( \text{Re} \) parts \( G_s(p^2), G_v(p^2), A(p^2), B(p^2) \)
Realistic K matrix

Dressed $\pi NN$ vertex

Dressed $N$ propagator including self-energy

Dressed $\pi NN$ vertex:

\[ p'^2 = m^2, \quad q^2 = m^2, \quad p^2 \neq m^2 \]

\[ \Gamma(p) = \gamma^5 \left\{ G_S(p^2) + \frac{\not{p} + m}{2m} G_V(p^2) \right\} \]

$G_S(p^2)$ and $G_V(p^2)$: half-off-shell pion-nucleon vertex functions

Nucleon self-energy:

\[ \Sigma(p) = A(p^2) \not{p} + B(p^2) m \]

$A(p^2)$ and $B(p^2)$: Self-energy functions
Dressed $\pi NN$-vertex and $N$-propagator

$G^0_{S,V}(p^2)$, bare form factor
$G_{S,V}(p^2)$, final form factor, free propagator
$G_{\Sigma,S,V}(p^2)$, final form factor, dressed propagator

- Convergence $\Rightarrow$ Upper limit width of bare f.f.
- Softening of the form factors due to the dressing
- Multi-loop corrections are large
- Form factors depend on the representation
Dressing the $\gamma N N$ vertex

Current conservation (gauge invariance)

- For Compton amplitude:
  \[ q'_\mu T^{\mu\nu} = q_\nu T^{\mu\nu} = 0 \]

- For $\gamma NN$ vertex (Ward-Takahashi identity):
  \[ (p'^\mu - p^\mu) \Gamma_\mu = \phi' - \phi - \Sigma(p') + \Sigma(p) \]

\[ \Rightarrow \begin{cases} 
F_{1\pm}^\pm [\Sigma] \\
F_{2\pm}^\pm 
\end{cases} \]

constrained by gauge inv.
not constrained by gauge inv.

$\gamma \pi NN$ contact term, built by minimal substitution
(transverse part ambiguous)

\[ p_i \rightarrow p_\mu \rightarrow p_\mu - e A_\mu \rightarrow \ldots \]
Including the photon

Coupled-channel K-matrix approach to $\gamma N \rightarrow \gamma N$, $\gamma N \rightarrow \pi N$ and $\pi N \rightarrow \pi N$

K matrix for Compton scattering

Dressed $\gamma NN$ vertex:

\[
\begin{align*}
\Gamma_\mu(p) &= \sum_{l=\pm} \left\{ \gamma_\mu F_1^l(p^2) + i \frac{\sigma_{\mu\nu}q^\nu}{2m} F_2^l(p^2) \right\} \Lambda_l(p) \\
F_1^\pm(p^2) \text{ and } F_2^\pm(p^2): &\text{ half-off-shell form factors}
\end{align*}
\]
Dressed $\gamma NN F_2$ form factors

Use of dispersion relations
⇒ Characteristic shape of the form factors

Pion photoproduction multipoles

Data: R.A. Arndt et al., PRC53, 430 (1996)
p(γ, π)p ; Partial Wave Amplitudes [m fm]

Photon lab energy [MeV]

Proton lab energy [MeV]

N(γ, π)/N proton channel

N(γ, π)/N A-channel

pion-production

Dressed K, model 11

May 17, 2002

GSI-93
Compton scattering

\( \gamma \gamma NN \) contact term

Built by minimal substitution

Additional contribution: plays a role of the "handbag" diagram; built using dispersion relations

Loop saturated by \( J^{\pi} = 1/2^- \) partial wave at threshold

Structure thus:

\[
F_{cnt}(p'^2) \propto \frac{\sigma^{\mu\rho} q_{\rho}}{2m} \Lambda_- \frac{\sigma^{\nu\sigma} q_{\sigma}}{2m}
\]

with

\[
F_{cnt}(p'^2) = \int \frac{d p'^2}{p'^2 - p^2} \frac{(ImF_2^- (p'^2))^2}{Tr (\Lambda_- \Sigma(p'))}
\]
Compton scattering cross sections

\[ \frac{d\sigma(\gamma, \gamma')}{d\Omega} \text{ [nb/sr]} \]

- \( E_{\text{lab}} = 149 \) MeV
- \( E_{\text{lab}} = 182 \) MeV
- \( E_{\text{lab}} = 230 \) MeV
- \( E_{\text{lab}} = 286 \) MeV

\[ \theta_{\text{cm}} \text{ [deg]} \]

\[ \theta_{\text{cm}} = 75^\circ \]

\[ \theta_{\text{cm}} = 90^\circ \]

GLE_G3s
Tue Nov 7 2000
Cusp of Compton amplitude at the pion threshold

- $f_{EE}^{1-}$ is related to the s-wave pion photoproduction
- Unitarity and causality are crucial for the cusp

Bare
Dressed
Nucleon polarizabilities
Compton cross section at low energies:

\[ \sigma(\omega) = c_0 + c_1 \omega + c_2 \omega^2 + c_3 \omega^3 + \ldots \]

\( c_{0,1}(m, e, \kappa) \) Low-energy theorem; model-independent
\( c_2 \sim \alpha, \beta \) Electric and magnetic polarizabilities
\( c_3 \sim \gamma \) Spin polarizabilities

Polarizabilities:
deformation of the nucleon in an external e.m. field

<table>
<thead>
<tr>
<th>Proton</th>
<th>F</th>
<th>B</th>
<th>( \chi PT )</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Gel00</td>
<td>Hem98</td>
<td>DA</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>12.1</td>
<td>15.5</td>
<td>10.5</td>
<td>16.4</td>
<td>11.9</td>
</tr>
<tr>
<td>( \beta )</td>
<td>2.4</td>
<td>1.7</td>
<td>3.5</td>
<td>9.1</td>
<td>1.9</td>
</tr>
<tr>
<td>( \gamma_{E1} )</td>
<td>-5.0</td>
<td>-1.7</td>
<td>-1.9</td>
<td>-5.4</td>
<td>-4.3</td>
</tr>
<tr>
<td>( \gamma_{M1} )</td>
<td>3.4</td>
<td>3.8</td>
<td>0.4</td>
<td>1.4</td>
<td>2.9</td>
</tr>
<tr>
<td>( \gamma_{E2} )</td>
<td>1.1</td>
<td>1.0</td>
<td>1.9</td>
<td>1.0</td>
<td>2.2</td>
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<tr>
<td>( \gamma_{M2} )</td>
<td>-1.8</td>
<td>-2.3</td>
<td>0.7</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>2.4</td>
<td>-0.9</td>
<td>-1.1</td>
<td>2.0</td>
<td>-0.8</td>
</tr>
<tr>
<td>( \gamma_\pi )</td>
<td>11.4</td>
<td>8.9</td>
<td>3.5</td>
<td>6.8</td>
<td>9.4</td>
</tr>
</tbody>
</table>

F: Full
B: Bare
Gel00: NLO
Hem98: \( \mathcal{O}(\epsilon^3) \)
DA: Dispersion Analyses
Sum rule for the forward spin polarisability
Dispersion analysis: $\gamma_0 = -0.8$

Baldin-Lapidus sum rule $(\alpha + \beta)$
Dispersion analysis: $11.9 + 1.9 = 13.8$
Sumrules
Gerasimov Drell-Hearn
\[ \frac{m^2}{2\pi e^2} \int_{\omega_{th}}^{\infty} d\omega \frac{\sigma_{1/2} - \sigma_{3/2}}{\omega} = -\frac{\kappa^2}{4} \]

sum rule for the forward spin polarisability
\[ \frac{1}{4\pi^2} \int_{\omega_{th}}^{\infty} d\omega \frac{\sigma_{1/2} - \sigma_{3/2}}{\omega^3} = \gamma_0 \]

Baldin Lapidus:
\[ \frac{1}{4\pi^2} \int_{\omega_{th}}^{\infty} d\omega \frac{\sigma_{1/2} + \sigma_{3/2}}{\omega^2} = \alpha + \beta \]

proton Results
\( \alpha + \beta \) in \([10^{-4} \text{ fm}^3]\), \( \gamma_0 \) in \([10^{-4} \text{ fm}^4]\).

<table>
<thead>
<tr>
<th></th>
<th>(\alpha + \beta)</th>
<th>(\alpha + \beta)</th>
<th>(\gamma)</th>
<th>(\gamma)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>low E</td>
<td>sum R</td>
<td>low E</td>
<td>sum R</td>
</tr>
<tr>
<td>Bare</td>
<td>15.8 + 1.4 = 17.2</td>
<td>14.5</td>
<td>-0.9</td>
<td>-1.2</td>
</tr>
<tr>
<td>part. dress</td>
<td>8.9 + 2.4 = 11.3</td>
<td>13.8</td>
<td>-0.1</td>
<td>-0.7</td>
</tr>
<tr>
<td>Full</td>
<td>12.1 + 2.4 = 14.5</td>
<td>13.8</td>
<td>2.4</td>
<td>-0.7</td>
</tr>
<tr>
<td>Full ((\Delta \text{ mod}))</td>
<td>12.1 + 1.6 = 13.7</td>
<td>13.8</td>
<td>-0.7</td>
<td>-0.7</td>
</tr>
</tbody>
</table>
Summary

- Dressing procedure in a K-matrix approach
  - Unitarity, crossing symmetry, gauge invariance satisfied
  - Analyticity properties of vertices and propagators implemented

- Coupled-channel description of pion-nucleon scattering, pion photoproduction and Compton scattering at low and intermediate energies.

- Causality (analyticity) constraints are important for low-energy Compton scattering
  - Cusp at the pion production threshold
  - Nucleon polarizabilities
    (electric, magnetic and spin)

- Multi-loop corrections can be large